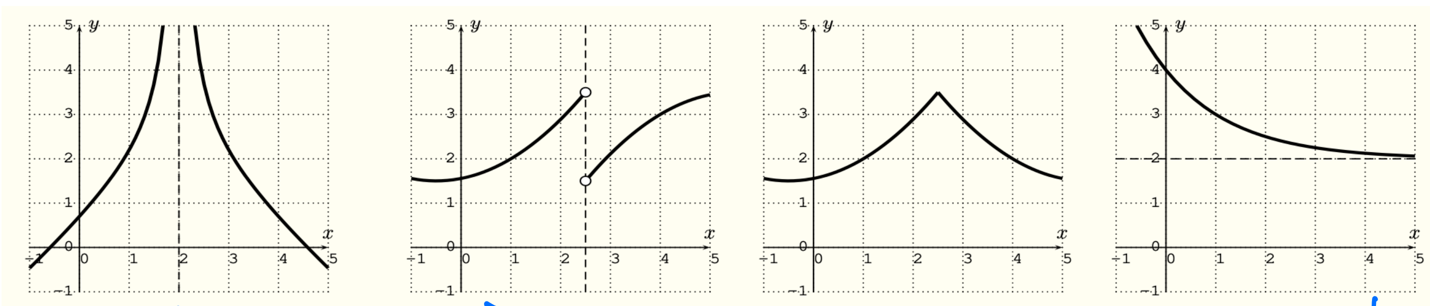
MAT 1375, Classwork8, Fall2024

ID:____________________ Name: Sol.

1. The domain of a polynomial f is all real number, and it is continuous for all real numbers and there are no jumps, no Vertical or horizontal asymptotes, and no corner/cusp

The following graphs **cannot** be graphs of polynomials:



Vertical asymptote/pole — jump — corner/cusp — horizontal asymptote

2. Factors and roots of polynomials:

Every n-degree polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x^1 + a_0$, $(a_n \neq 0)$ can be factored as

$$f(x) = a_n(x - \boldsymbol{c_1})(x - \boldsymbol{c_2})\cdots(x - \boldsymbol{c_n}).$$

Thus, the polynomial $f(x)$ of degree n has **at most** n roots (which are $c_1, c_2, \cdots, c_n$) and these roots may be either real or complex

Let f be a polynomial with all **real coefficients**. The complex roots are always found as a **pair**, that is, if $c = a + bi$ is a complex root of f, then the complex conjugate $\bar{c} =$ $a - b\bar{i}$ is also a root of f.

3. Let $f(x) = x^3 - x^2 + 2$. Find all the roots of $f(x)$. Sketch a complete graph and label all roots.

Sol ① Guess Roots: $f(-1)=0$

$\Rightarrow (x+1)$ is a factor

② Long division

$$\begin{array}{r|l} & 1 \ -2 \ +2 \\ \hline 1 \ +1 & 1 \ -1 \ +0 \ +2 \\ & -(1+1) \\ \hline & \quad -2 \ +0 \\ & \quad -(-2 \ -2) \\ \hline & \qquad 2+2 \\ & \qquad -(2+2) \\ \hline & \qquad\quad 0 \end{array}$$

③ $x^3 - x^2 + 2 = (x+1)(x^2-2x+2) = 0$

$\Rightarrow (x+1)=0$ or $x^2-2x+2=0$

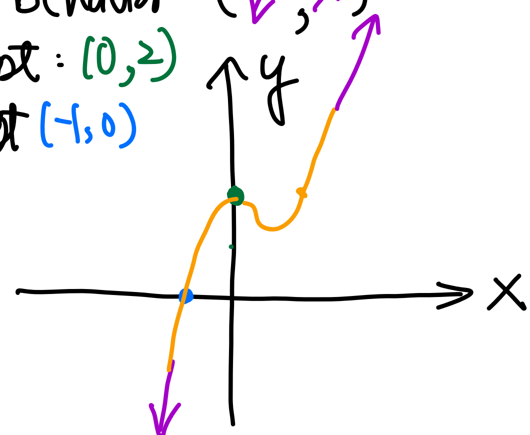
$\Rightarrow x=-1$ or $x = \dfrac{2 \pm \sqrt{-4}}{2} = 1 \pm i$

The quadratic formula

④ End behavior $(\swarrow, \nearrow)$

y-intercept: $(0,2)$

x-intercept $(-1,0)$



4. Definition of the **Rational function**:

A rational function is a fraction of two polynomials $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are both polynomials, and $q(x) \neq 0$.

The **domain of a rational function** f is all real numbers for which the denominator $q(x)$ is not zero: $D_f = \{ x \mid q(x) \neq 0 \}$

$x \in \mathbb{R}$

5. **Arrow Notation**: Given a constant a and we have

$x \to a^+$:	x approaches a from the **right** (x is very closed to a but $x \neq a$ and x > a)
$x \to a^-$:	x approaches a from the **left** (x is very closed to a but $x \neq a$ and x < a)
$x \to \infty$:	x approaches infinity (x increases without bound)
$x \to -\infty$:	x approaches negative infinity (x decreases without bound)