MAT 1375, Classwork7, Fall2024

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1. Definition of **Polynomial function of degree n** in one variable:

A Polynomial of degree n in one variable is a function f of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x^1 + a_0,$$

for some constants $a_0, a_1, \cdots, a_n$, where $a_n \neq 0$ and n is a non-negative integer. The numbers $a_0, a_1, \cdots, a_n$ are called coefficients.

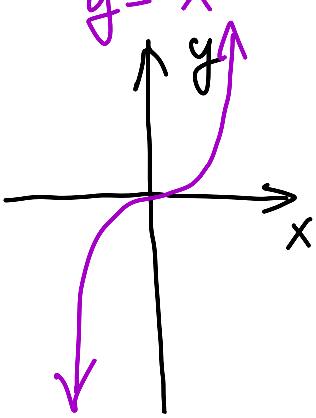
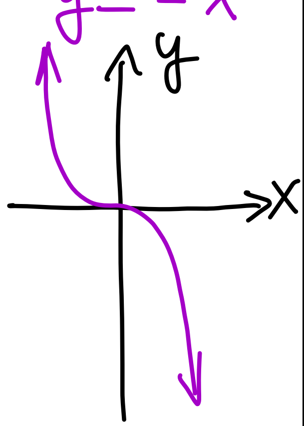
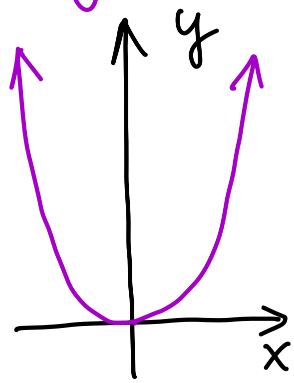
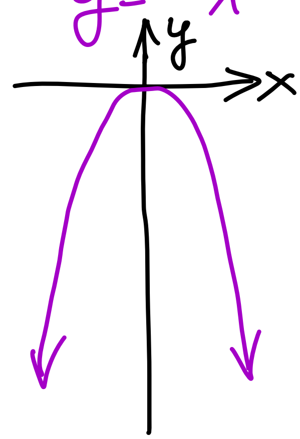
The **number a_n**, the coefficient of the variable to the highest power, is called the leading coefficient and n is the degree of the polynomial.

2. The **End Behavior of the polynomials** and the **Leading Coefficient Test**:

As x goes to ∞ or $-\infty$, the graph of polynomial function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x^1 + a_0, \qquad (a_n \neq 0)$$

either **rises** or **falls** eventually. Here, we can conclude this into the following table

n is an **odd** number		n is an **even** number	
$a_n > 0$	$a_n < 0$	$a_n > 0$	$a_n < 0$
($\swarrow$, $\nearrow$)	($\nwarrow$, $\searrow$)	($\nwarrow$, $\nearrow$)	($\swarrow$, $\searrow$)
example $y = x^3$ 	$y = -x^3$ 	$y = x^2$ 	$y = -x^2$ 

3. A root or zero or solution of a polynomial $f(x)$ is **a number c** so that $f(c) =$ 0. Each **real** root/zero/solution of the polynomial $f(x)$ appears as an x-intercept of the graph of $f(x)$. (Here `**real**' means not a complex number)

4. **Multiplicity** of the root and x-Intercepts:

Let $f(x) = (x - r)^k$ where r is the root of f and this root repeats K times. We call r a root with multiplicity k.

Even Multiplicity (k is even)	Odd Multiplicity (k is odd)
The graph touches the x-axis and turns around at the root r.	The graph crosses the x-axis at the root r.
The graph tends to **flatten out** near the roots with **multiplicity** greater than 1	

5. **Turning Points** of Polynomial Functions:

Let $f(x)$ be a polynomial function of **degree n**, then the graph of f has at most n-1 turning points.

6. The essential part for drawing **a complete graph of f**:

- long-range End Behavior by leading coefficient test (how the function behaves when X approaches $\pm\infty$)
- All roots (which are X - intercepts) with the Multiplicities
- All y-intercepts (the values by computing f(0))
- All asymptotes (for rational functions in next chapter)
- Turning points with Extrema (that is all maxima and minima)