MAT 1375, Classwork6, Fall2024

ID:_____ Name:_____

1. Let f and g be the functions defined by the table below. Complete the table by performing the indicated operations.

x	1	2	3	4	5	6	7
$f(x)$	4	5	7	0	-2	6	4
$g(x)$	6	-8	5	2	9	11	2
$g(x)+3$	9	−5	8	5	11	14	5
$f(x)-2g(x)$	−8	21	−3	−4	−20	−16	0
$g(x+3)$	2	9	11	2	undefined	undefined	undefined
$(f\circ g)(x)$	11	undefined	−2	5	undefined	undefined	−8
$(g\circ f)(x)$	2	9	2	undefined	undefined	11	2
$(g\circ g)(x)$	11	undefined	9	−8	undefined	undefined	−8

2. Complete the definition of the **one-to-one function** (or **injective**):

Given a function $f(x)$. If any two different inputs $x_1 \neq x_2$ always have different outputs $f(x_1) \neq f(x_2)$, then we call this function f a **one-to-one function**.

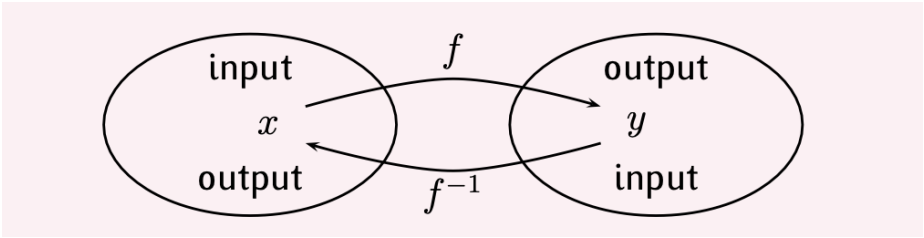
3. **Horizontal Line test**:

A function is one-to-one when every horizontal line intersects the graph of the function at most once.

4. Complete the definition of **the Inverse of a Function**:

Let f be a function with domain D_f and the range R_f, and assume that f is one-to-one. The **inverse** of f is the function f^{-1}, determined by:

$f(x) = y$ means precisely that $f^{-1}(y) = x$



Therefore, we have $D_{f^{-1}} =$ R_f, and $R_{f^{-1}} =$ D_f.

5. How to check if two given functions are **inverse** with each other:

   Let f and g be two functions such that

   $f(g(x)) = x$ for every x in the domain of g **and**

   $g(f(x)) = x$ for every x in the domain of f.

   The function g is the **inverse of the function f** and is denoted by $g = f^{-1}$.

6. How to find the inverse function for a given **invertible** function $f(x)$:

   Step1: Replace $f(x)$ with y

   Step2: Interchang x and y

   Step3: Solve for y (isolate y)

   Step4: Replace y with $f^{-1}(x)$

x	0	1	2	3
$f(x)$	1	2	5	10

7. Given a function $f(x) = x^2 + 1, x \geq 0$. a) Find the inverse function of $f(x)$. b) Graph f and f^{-1} in the same coordinate system.

Sol

a) step1: $y = x^2 + 1, \quad x \geq 0$

step2: $x = y^2 + 1, \quad y \geq 0$

step3: $y^2 = x - 1, \quad y \geq 0$

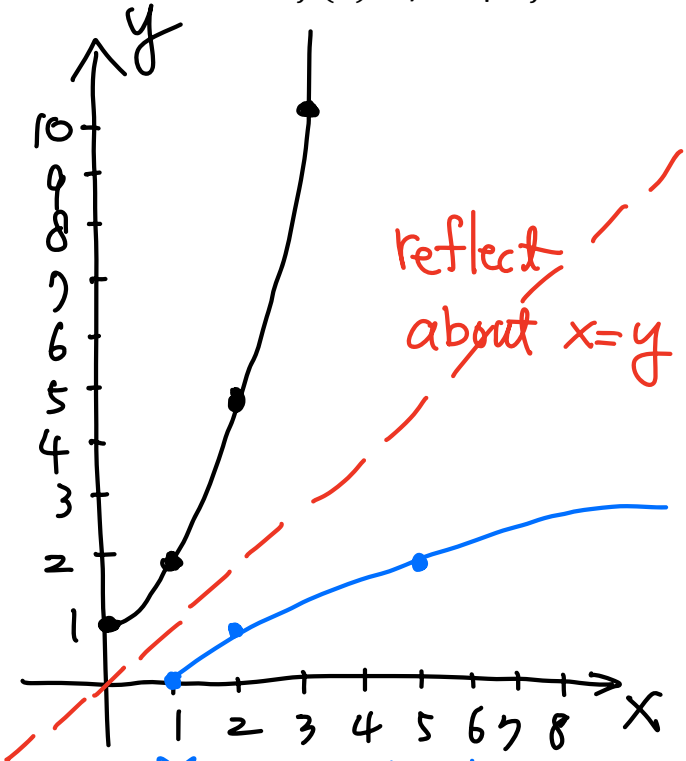
$y = \pm\sqrt{x-1}, \quad x \geq 1$

$y \geq 0$

no "−" case since $y \geq 0$ $\Rightarrow y = \sqrt{x-1}$

step4: $f^{-1}(x) = \sqrt{x-1}, \quad x \geq 1$

b)



x	1	2	5	10
$f^{-1}(x)$	0	1	2	3