MAT 1375, Classwork5, Fall2024

ID:_____ Name:_____

1. Complete the definition of **the Algebra of Functions**:

Let $f(x)$ and $g(x)$ be two functions with the domain D_f and D_g ,respectively. We have sum, difference, product, and quotient of functions:

The Algebra of functions	Notation	Definition	Domain
Sum	$(f+g)(x) :=$	$f(x)+g(x)$	$D_{f+g} = D_f \cap D_g$
Difference	$(f-g)(x) :=$	$f(x)-g(x)$	$D_{f-g} = D_f \cap D_g$
Product	$(f\cdot g)(x) :=$	$f(x)\cdot g(x)$	$D_{f\cdot g} = D_f \cap D_g$
Quotient	$(\frac{f}{g})(x) :=$	$\frac{f(x)}{g(x)}$,provided $g(x)\neq 0$	$D_{\frac{f}{g}} = D_f \cap D_g$ and $g(x)\neq 0$

Here, $D_f \cap D_g = \{ x \mid$ $x\in D_f$ and $x\in D_g$ $\}$

2. Let $f(x) = x^2+5x+6$ and $g(x) = x+2$. Find the following functions and state their domains.

$D_f = (-\infty,\infty)$, $D_g = (-\infty,\infty)$ domain

$(f+g)(x) = (x^2+5x+6)+(x+2) = x^2+6x+8$ $\quad D_f \cap D_g = (-\infty,\infty)$

$(f-g)(x) = (x^2+5x+6)-(x+2) = x^2+4x+4$ $\quad D_f \cap D_g = (-\infty,\infty)$

$(f\cdot g)(x) = (x^2+5x+6)\cdot(x+2) = x^3+7x^2+16x+12$ $\quad D_f \cap D_g = (-\infty,\infty)$

$\left(\frac{f}{g}\right)(x) =$

	x^2+5x+6
x	x^3 | $5x^2$ | $6x$
$+2$	$2x^2$ | $10x$ | 12

$\frac{x^2+5x+6}{x+2}$ $\quad D_f \cap D_g$ but $g(x)\neq 0$

$\Rightarrow (-\infty,\infty)$ but $x+2\neq 0$

$\Rightarrow (-\infty,\infty)$ $x\neq -2$

$(-\infty,-2)\cup(-2,\infty)$

$\left(= \frac{(x+2)(x+3)}{(x+2)} = \frac{?}{1} (x+3)\right)$ This is true only when $x\neq -2$

3. Complete the definition of **the Composition of Functions**:

Let $f(x)$ and $g(x)$ be two functions. The composition of the function f with g is denoted by $(f\circ g)(x)$ and is defined by the equation

$$(f\circ g)(x) := f(g(x)).$$

The domain of the composition of the function $f \circ g$ is the set of all x such that x is the input of $g(x)$, and $g(x)$ is the domain of $f(x)$.

The notation of the domain of the composition of the function $f \circ g$ is

$$D_{f\circ g} = \{ x \mid x \in D_g \text{ and } g(x) \in D_f \}$$

4. Find $(f \circ g)(x)$ for the following functions and state their domains.

a) $f(x) = x^2 + 2$ and $g(x) = x - 3$

Sol $(f\circ g)(x) = f(g(x))$

$= f(x-3) = (x-3)^2+2$

$= x^2-6x+9+2 = x^2-6x+11$

Domain

$D_{f\circ g} = (-\infty, \infty)$

(since $D_g = (-\infty,\infty)$ and there is no restriction from $f(x)$)

b) $f(x) = \frac{2}{x-3}$ and $g(x) = x^2 + 2x$

Sol $(f\circ g)(x) = f(g(x))$

$= f(x^2+2x)$

$= \frac{2}{(x^2+2x)-3} = \frac{2}{x^2+2x-3}$

Domain

$D_{f\circ g} = (-\infty,-3)\cup(-3,1)\cup(1,\infty)$

(We start it from $D_g = (-\infty,\infty)$. Since f has a restriction, as a fraction, its denominator can't be zero, thus two points are excluded from the domain)

$x^2+2x-3 \neq 0 \Rightarrow (x+3)(x-1)\neq 0$

$\Rightarrow x+3\neq 0$ and $x-1\neq 0 \Rightarrow x\neq -3$ and $x\neq 1$