MAT 1375, Classwork16, Fall2024

ID:________________________________ Name:________________________________

1. Addition and Subtraction of angles formulas:

Let α, β be two angles. We have

$$(1)\ \sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$(2)\ \sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$(3)\ \cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$(4)\ \cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

2. Half- and double-angle formulas:

$$(5)\ \sin(2\alpha) = 2\sin(\alpha)\cos(\alpha) \text{ (From (1) and let } \beta = \alpha)$$

$$(6)\ \cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \text{ (From (3) and let } \beta = \alpha)$$

$$(7)\ \sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{2}} \quad \big((4)-(3) \big)$$

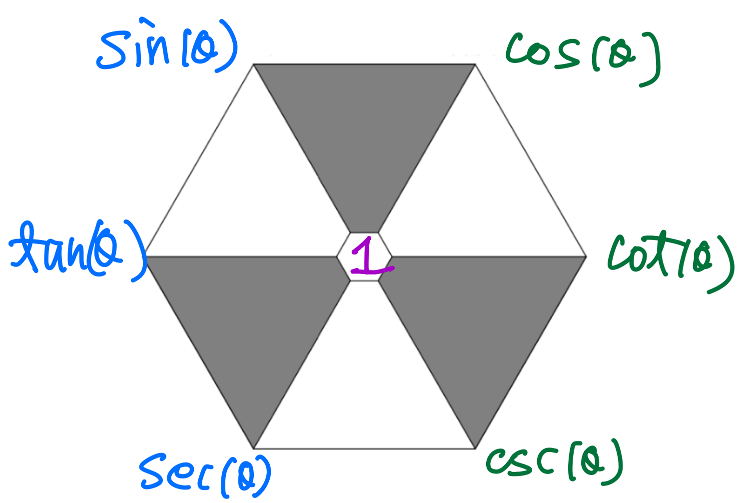
$$(8)\ \cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}} \quad \big((4)+(3) \big)$$

3. Pythagorean Identities:

$$(9)\quad \sin^2(\theta) + \cos^2(\theta) = 1^2$$

$$(10)\quad 1^2 + \tan^2(\theta) = \sec^2(\theta)$$

$$(11)\quad 1^2 + \cot^2(\theta) = \csc^2(\theta)$$



$30^\circ\left(\frac{\pi}{6}\right) \quad 45^\circ\left(\frac{\pi}{4}\right) \quad 60^\circ\left(\frac{\pi}{3}\right)$

4. Find the exact value of the trigonometric functions:

a) $\sin\left(\frac{11\pi}{12}\right)$ b) $\cos\left(\frac{7\pi}{8}\right)$ c) $\sin(15^\circ)$ d) $\cos(75^\circ)$

d) $\cos(75^\circ) = \cos(30^\circ+45^\circ) = \cos(30^\circ)\cos(45^\circ) - \sin(30^\circ)\sin(45^\circ)$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

c) $\sin(15^\circ) = \sin(60^\circ-45^\circ) = \sin(60^\circ)\cos(45^\circ) - \cos(60^\circ)\sin(45^\circ)$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

a) $\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{2\pi}{3}+\frac{\pi}{4}\right) = \sin\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right)\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

b) $\cos\left(\frac{7\pi}{8}\right) = \cos\left(\frac{1}{2}\cdot\frac{7\pi}{4}\right)$

$= \cancel{+}-\sqrt{\frac{1+\cos\left(\frac{7\pi}{4}\right)}{2}} = -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = \boxed{-\sqrt{\frac{2-\sqrt{2}}{4}}}$

$\cos\left(\frac{7\pi}{8}\right) < 0$

5. Simplify the given function using the addition and subtraction formulas.

a) $\sin\left(\frac{\pi}{2}+x\right)$ b) $\sin\left(\frac{\pi}{2}-x\right)$ c) $\cos\left(\frac{\pi}{2}+x\right)$ d) $\cos\left(\frac{\pi}{2}-x\right)$

a) $\sin\left(\frac{\pi}{2}+x\right) = \sin\left(\frac{\pi}{2}\right)\cos(x) + \cos\left(\frac{\pi}{2}\right)\sin(x)$

$= 1\cdot\cos(x) + 0\cdot\sin(x) = \boxed{\cos(x)}$

b) $\sin\left(\frac{\pi}{2}-x\right) = \sin\left(\frac{\pi}{2}\right)\cdot\cos(x) - \cos\left(\frac{\pi}{2}\right)\cdot\sin(x)$

$= 1\cdot\cos(x) - 0\cdot\sin(x) = \boxed{\cos(x)}$

c) $\cos\left(\frac{\pi}{2}+x\right) = \cos\left(\frac{\pi}{2}\right)\cos(x) - \sin\left(\frac{\pi}{2}\right)\cdot\sin(x)$

$= 0\cdot\cos(x) - 1\cdot\sin(x) = \boxed{-\sin(x)}$

d) $\cos\left(\frac{\pi}{2}-x\right) = \cos\left(\frac{\pi}{2}\right)\cdot\cos(x) + \sin\left(\frac{\pi}{2}\right)\cdot\sin(x)$

$= 0\cdot\cos(x) + 1\cdot\sin(x) = \boxed{\sin(x)}$