MAT 1375, Classwork14, Fall2024

ID:____________________________________ Name:____________________________________

1. The function to describe the population growths or decline:

$$f(t) = c \cdot b^t, \quad t\text{: time}$$

where $f(0) =$ __c__ which is initial condition at $t =$ __0__, and $b > 0, b \neq 1$.

If the **rate of growth** r is given, then the base $b =$ __e^r__ and $f(t) = c \cdot (e^r)^t. = c \cdot e^{rt}$

2. The population of a country grows exponentially at a rate of 2.6% per year. If the population was 35.7 million in the year 2000, then what is the population size of this country in the year 2027?

$r = 0.026$ $c = 35.7$ $t = 0$ $t = 27$

$f(t) = c \cdot e^{rt} \Rightarrow f(t) = 35.7 \cdot e^{0.026t}$

$\Rightarrow f(27) = 35.7 e^{(0.026)(27)}$ million.

3. How much do you get if you invest $500 today at 3% compounded quarterly in 3 years?

$c = 500$ $r = 0.03$ $n = 4$ $t = 3$

$500 \cdot \left(1 + \frac{0.03}{4}\right)^{4 \cdot 3}$

4. If a principal (i.e. initial amount) ***P*** is invested for ***t*** years at a rate ***r*** and compounded ***n*** times per year, then the final amount ***A*** is given by

$$A(t) = P \cdot \left(1 + \frac{r}{n}\right)^{n \cdot t}.$$

If a **continuous compounding** ($n \to$ __∞__) is considered, then the final amount ***A*** with a continuous compounding is given by $A(t) = P \cdot (e^r)^t$.

5. The exponential function $f(t)$ has a half-life of ***h*** if the base is given by

$$b = \left(\frac{1}{2}\right)^{\frac{1}{h}},$$

and $f(t) = c \cdot \left(\left(\frac{1}{2}\right)^{\frac{1}{h}}\right)^t =$ __$c \cdot \left(\frac{1}{2}\right)^{\frac{t}{h}}$__.