MAT 1375, Classwork13, Fall2024

ID:________________________ Name:________________________

1. The Exponential and Logarithmic functions and one-to-one property:

For $b > 0, b \neq 1$, the exponential and logarithmic functions are one-to-one:

$$b^x = b^y \iff x = y$$

$$\log_b(x) = \log_b(y) \iff x = y$$

2. Solve for x:

$4 \cdot \log_2(2)$

a) $\log_2(x+5) = \log_2(x+3) + 4$. b) $\log(x) + \log(x+4) = \log(5)$.

c) $\ln(x+2) + \ln(x-3) = \ln(7)$. d) $\log_5(x-7) + \log_5(2-x) = \log_5(4)$.

Sol: a) $\log_2(x+5) = \log_2(x+3) + \log_2(2^4)$ power rule

Product Rule

$\log_2(x+5) = \log_2(2^4 \cdot (x+3))$

$\iff x+5 = 2^4(x+3)$

$\Rightarrow x+5 = 16x+48$

$\Rightarrow 15x = -43$

$\Rightarrow x = \frac{-43}{15}$

(since $x+5>0$, $x+3>0$, then $x = -\frac{43}{15}$ is an answer)

b) $\log(x) + \log(x+4) = \log(5)$

product rule

$\log x \cdot (x+4) = \log(5)$

$\iff x(x+4) = 5$

$\Rightarrow x^2+4x = 5$

$\Rightarrow x^2+4x-5 = 0$

$\Rightarrow (x-1)(x+5) = 0$

$x = 1$ or ~~$x = -5$~~

since, for $\log(x)$, $x > 0$, then x cannot be -5

3. Solve for x:

a) $2^{x+7} = 32$. b) $10^{2x-8} = 0.01$. c) $27^{x+3} = 9^{x-1}$. d) $8^{x+2} = 4^{x-3}$.

a) $2^{x+7} = 2^5$

$\iff x+7 = 5$

$\Rightarrow x = -2$

b) $10^{2x-8} = 10^{-2}$

$(0.01 = \frac{1}{100} = 10^{-2})$

$\iff 2x-8 = -2$

$\Rightarrow 2x = 6$

$\Rightarrow x = 3$

c) $(3^3)^{(x+3)} = (3^2)^{(x-1)}$

$(27 = 3^3,\ 9 = 3^2)$

$\Rightarrow 3^{3(x+3)} = 3^{2(x-1)}$

$\iff 3(x+3) = 2(x-1)$

$\Rightarrow 3x+9 = 2x-2$

$\Rightarrow x = -11$

d) $(2^3)^{(x+2)} = (2^2)^{(x-3)}$

$\Rightarrow 2^{3(x+2)} = 2^{2(x-3)}$

$\iff 3(x+2) = 2(x-3)$

$\Rightarrow 3x+6 = 2x-6$

$\Rightarrow x = -12$

2. Solve for x.

c) $\ln(x+2)+\ln(x-3)=\ln(7)$. d) $\log_5(x-7)+\log_5(2-x)=\log_5(4)$

Sol (c)

$$\ln(x+2)+\ln(x-3)=\ln(7)$$

product rule

$$\ln(x+2)\cdot(x-3)=\ln(7)$$

$$\Leftrightarrow (x+2)(x-3)=7 \Rightarrow x^2-x-6=7$$

$$\Rightarrow x^2-x-13=0 \Rightarrow x=\frac{13\pm\sqrt{(-1)^2-4\cdot1\cdot(-13)}}{2}=\frac{13\pm\sqrt{53}}{2}$$

$$\Rightarrow x=\frac{13+\sqrt{53}}{2} \quad \text{or} \quad \cancel{x=\frac{13-\sqrt{53}}{2}}$$

(since, for $\ln(x-3)$, $x>3$ but $\frac{13-\sqrt{53}}{2}<3$

So, $x\neq\frac{13-\sqrt{53}}{2}$)

d) $\log_5(x-7)+\log_5(2-x)=\log_5(4)$

product rule

$$\log_5(x-7)\cdot(2-x)=\log_5(4) \Leftrightarrow (x+7)(2-x)=4$$

$$\Rightarrow -x^2+2x+7x-14=4 \Rightarrow x^2-9x+18=0$$

x -3

x -6

$$\Rightarrow (x-3)(x-6)=0 \Rightarrow \cancel{x=3} \text{ or } \cancel{x=6}.$$ NO solution

(since, for $\log_5(x-7)$, x has to be more than 7, then $x=3$, $x=6$ are not the answers)

4. How about the equations **without** the same base on both sides?(For example, $4^x = 15$.)

5. Using **Logarithms** to Solve **Exponential Equations**:

Step1: Isolate the exponential expression.

Step2: Take logarithm on both sides of the equation for common base 10 or natural base e.

Step3: Simplify using one of these properties:

$\ln(b^x) = x \cdot \ln(b)$ (if $b \neq 10$) or $\ln(e^x) = x \cdot 1$ ($\ln(e)$) or $\log(10^x) = x \cdot 1$ ($\log(10)$).

Step4: Solve for x.

6. Solve for x.

a) $3^{x+5} = 8$. b) $13^{2x-4} = 6$. c) $5.1^x = 2.7^{2x+6}$. d) $7e^{2x} - 5 = 58$. e) $10^x = 800$.

Sol: a)

$$3^{x+5} = 8 \quad (\text{Bases are different})$$

Take "ln" on both sides:

$$\ln(3^{x+5}) = \ln(8)$$

$$(x+5) \cdot \ln(3) = \ln(8)$$

(power rule)

Solve for x:

$$x \cdot \ln(3) + 5 \cdot \ln(3) = \ln(8)$$

$$\Rightarrow x \cdot \ln(3) = \ln(8) - 5\ln(3)$$

$$\Rightarrow x = \frac{\ln(8) - 5\ln(3)}{\ln(3)}$$

b)

$$13^{2x-4} = 6 \quad (\text{Bases are different})$$

① Take "ln" on both sides:

$$\ln(13^{2x-4}) = \ln(6)$$

power rule

$$(2x-4) \cdot \ln(13) = \ln(6)$$

② Solve for x:

$$\Rightarrow 2x\cdot\ln(13) - 4\cdot\ln(13) = \ln(6)$$

$$\Rightarrow 2x\cdot\ln(13) = \ln(6) + 4\ln(13)$$

divided by $2\cdot\ln(13)$

$$\Rightarrow x = \frac{\ln(6) + 4\cdot\ln(13)}{2\ln(13)}$$

c) $5.1^{x} = 2.7^{2x+6}$ (Bases are different)

① Take "ln" on both sides:

$$\ln(5.1^{x}) = \ln(2.7^{2x+6})$$

power rule

$$x\cdot\ln(5.1) = (2x+6)\ln(2.7)$$

② Solve for x:

$$\Rightarrow x\cdot\ln(5.1) = 2x\cdot\ln(2.7) + 6\cdot\ln(2.7)$$

$$\Rightarrow x\ln(5.1) - 2x\ln(2.7) = 6\cdot\ln(2.7)$$

$$\Rightarrow x\left(\ln(5.1) - 2\cdot\ln(2.7)\right) = 6\cdot\ln(2.7)$$

$$\Rightarrow x = \frac{6\ln(2.7)}{\ln(5.1) - 2\ln(2.7)}$$

d) $7e^{2x} - 5 = 58$

$$\Rightarrow 7e^{2x} = 58 + 5 \Rightarrow \frac{7e^{2x}}{7} = \frac{63}{7}$$

$$\Rightarrow e^{2x} = 9$$ (Bases are different)

Take "ln" on both sides:

$$\ln(e^{2x}) = \ln(9)$$

(power rule)

$$\Rightarrow 2x\cdot\underbrace{\ln(e)}_{=1 \text{ (inverse property)}} = \ln(9)$$

$$\Rightarrow 2x = \ln(9) \xrightarrow[\text{by "2"}]{\text{divided}} x = \ln(9)$$

e) $10^x = 800$ (Bases are different)

Take "$\log_{10}$" on both sides: $\log(10^{x}) = \log(800)$

power rule

$x \cdot \underbrace{\log(10)}_{=1 \text{ (inverse property)}} = \log(800) \Rightarrow x = \log(800)$