1. Evaluate the expression by rewriting it as an exponential expression.

a) $\log_5(125)$. b) $\log_4(1)$. c) $\log_7\left(\frac{1}{49}\right)$. d) $\log_2(\sqrt[5]{2})$. e) $\log_{25}(5)$.

Sol: a) Let $x = \log_5(125)$. $\Rightarrow 5^x = 125 \Rightarrow x = 3$

b) Let $x = \log_4(1) \Rightarrow 4^x = 1 \Rightarrow x = 0$.

c) Let $x = \log_7\left(\frac{1}{49}\right) \Rightarrow 7^x = \frac{1}{49} = \frac{1}{7^2} = 7^{-2} \Rightarrow x = -2$

d) Let $x = \log_2(\sqrt[5]{2}) \Rightarrow 2^x = \sqrt[5]{2} = 2^{\frac{1}{5}} \Rightarrow x = \frac{1}{5}$

e) Let $x = \log_{25}(5) \Rightarrow 25^x = 5 = 5^{\frac{1}{2}} \Rightarrow x = \frac{1}{2}$

MAT 1375, Classwork12, Fall2024

ID:______________________________ Name:______________________________

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2. **Basic logarithmic evaluations**: Let $f(x) = b^x$ and $g(x) = \log_b x, b > 0, b \neq 1$. We have

Elementary logarithms: $b = b^1 \Leftrightarrow$ <u>1</u> $= \log_b(b)$.

$1 = b^0 \Leftrightarrow$ <u>0</u> $= \log_b(1)$.

Inverse properties: <u>$b^{g(x)} = b^{\log_b x}$</u> $= x$. $(f(g(x)) = x)$

<u>$\log_b(b^x)$</u> $= x$. $(g(f(x)) = x)$

Change-of-Base property: **10-base**: $\log_b(x) = \frac{\log_{10}(x)}{\log_{10}(b)} = \frac{\log(x)}{\log(b)}$.

natural base: $\log_b(x) = \frac{\log_e(x)}{\log_e(b)} = \frac{\ln(x)}{\ln(b)}$.

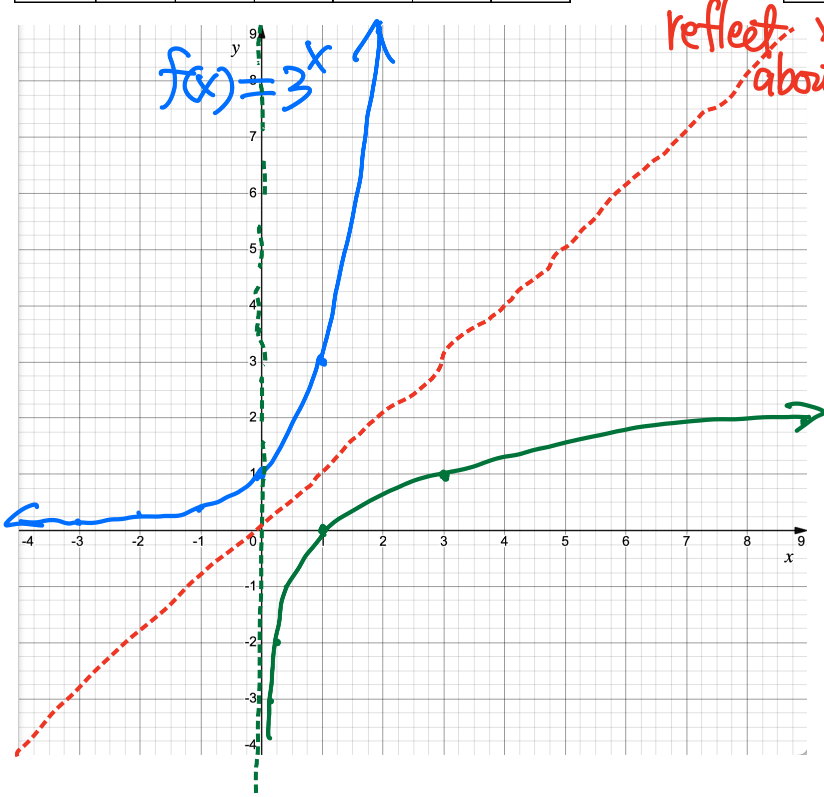
3. Graph $f(x) = 3^x$ and $g(x) = \log_3 x$ in the same coordinate.

inverse function

$(-\infty,\infty)$ x	-3	-2	-1	0	1	2
$(0,\infty)$ $f(x)$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

x	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
$g(x)$	-3	-2	-1	0	1	2

reflect about $x=y$



$-3 = \log_3 x \Rightarrow x =$ <u>$3^{-3} = \frac{1}{27}$</u>,

$-2 = \log_3 x \Rightarrow x =$ <u>$3^{-2} = \frac{1}{9}$</u>,

$-1 = \log_3 x \Rightarrow x =$ <u>$3^{-1} = \frac{1}{3}$</u>,

$0 = \log_3 x \Rightarrow x =$ <u>$3^0 = 1$</u>,

$1 = \log_3 x \Rightarrow x =$ <u>$3^1 = 3$</u>,

$2 = \log_3 x \Rightarrow x =$ <u>$3^2 = 9$</u>.

The domain of g: <u>$(0,\infty)$</u>.

The range of g: <u>$(-\infty,\infty)$</u>.

4. Characteristics of Exponential Function of $g(x) = \log_b x$. $b \neq 1$

(a) The domain of g: $\underline{(0,\infty)}$; The range of g: $\underline{(-\infty,\infty)}$.

(b) There is $\underline{\text{NO}}$ y-intercept. In fact, g approaches, but never touches $\underline{y\text{-axis}}$ $(x=0)$ which is a $\underline{\text{vertical}}$ asymptote of g.

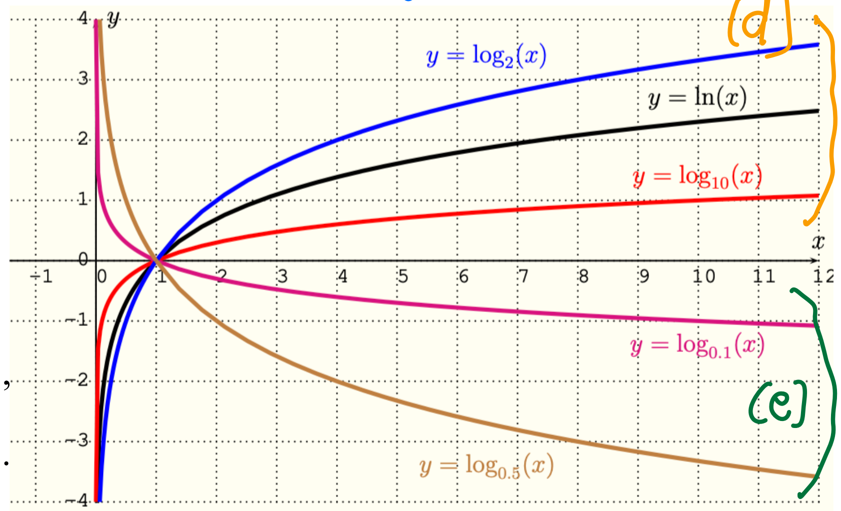
(c) Its x-intercept is $\underline{(1,0)}$.

(d) For $b > 1$, $g(x) \to \underline{\infty}$ as $x \to \infty$,

$g(x) \to \underline{-\infty}$ as $x \to 0^+$.

(e) For $0 < b < 1$, $g(x) \to \underline{-\infty}$ as $x \to \infty$,

$g(x) \to \underline{\infty}$ as $x \to 0^+$.



5. Properties of Logarithms: $\boxed{(\Delta = b^{\square} \Leftrightarrow \square = \log_b \Delta)}$ Let $X > 0, Y > 0, b > 0, b \neq 1$.

Exponential Identities	Logarithmic Identities
$b^x \cdot b^y = b^{x+y}$ (The same base multiplication = the addition of exponents)	$\log_b X + \log_b Y = \log_b(XY)$ (**The Product Rule:** The same log base addition = the product of the values)
$\frac{b^x}{b^y} = b^{x-y}$ (The same base division = the subtraction of exponents)	$\log_b X - \log_b Y = \log_b\left(\frac{X}{Y}\right)$ (**The Quotient Rule:** The same log base subtraction = the division of the values)
$(b^x)^n = b^{xn}$	$\log_b X^n = n \cdot \log_b X$ (**The Power Rule**)

The Product Rule: Let $X = b^x$, $Y = b^y$. We have

$x = \underline{\log_b X}$ and $y = \underline{\log_b Y}$.

Then $X \cdot Y = \boldsymbol{b^x \cdot b^y} = \boldsymbol{b^{x+y}} = b^{\log_b X + \log_b Y}$ implies

$\underline{\log_b X + \log_b Y} = \log_b(XY)$.

6. Combine the terms using the properties of logarithms to write as one logarithm.

a) $\frac{1}{2}\ln(x) + \ln(y)$. b) $5 + \log_2(a^2 - b^2) - \log_2(a+b)$

$= \ln x^{\frac{1}{2}} + \ln(y)$ (power rule) $= 5 \cdot 1 + \log_2\left(\frac{a^2-b^2}{a+b}\right)$ Quotient rule

$$= \ln\left(x^{\frac{1}{2}} \cdot y\right)$$

$$= 5 \cdot \log_2(2) + \log_2\left(\frac{a^2-b^2}{a+b}\right)$$

$$= \log_2\left(2^5\right) + \log_2\left(\frac{a^2-b^2}{a+b}\right)$$

(power rule)

product rule

$$= \log_2\left(2^5 \cdot \frac{a^2-b^2}{a+b}\right)$$

$(a+b)(a-b)$

$$= \log_2\left(2^5 \cdot (a-b)\right)$$