MAT 1375, Classwork11, Fall2024

ID:______________________ Name:______________________

1. Solve for x: (a) $|2x-3| \geq 7$. (b) $\frac{x^2-5x+6}{x^2-5x} \geq 0$

2. Definition of the **Exponential Function**:

A function f is called an exponential function with base b for **any real number x** if

$$f(x) = c \cdot b^x,$$

for some real number c and positive real number b which is called the base.

3. Please circle the given function if it is an **exponential function**:

e=2.71

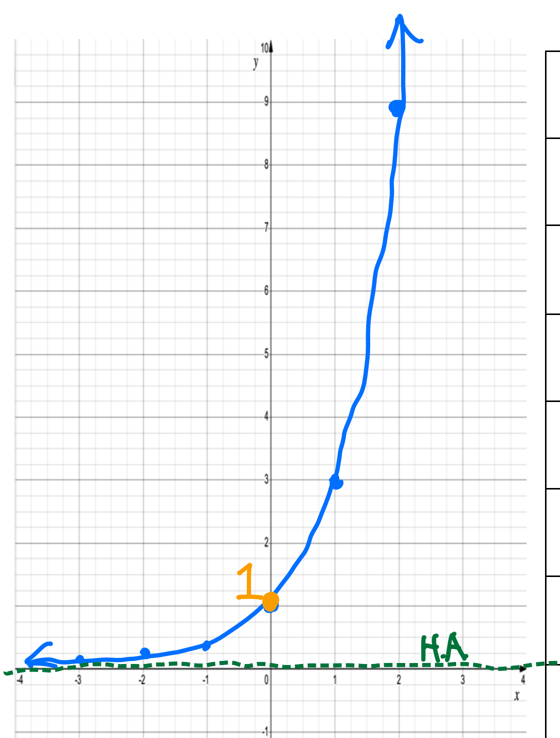
(1) $f(x) = 2^x$. (2) $g(x) = 3^{x+1}$. (3) $h(x) = e^x$. (4) $k(x) = \left(\frac{1}{5}\right)^x$. (5) $\cancel{l(x)} = x^2$

(6) $m(x) \cancel{= (-1)}^x$. (7) $n\cancel{(x) =} x^x$.

4. Graph the given functions:

(a) $f(x) = 3^x$.

x	$f(x)$
-3	$\frac{1}{27}$
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

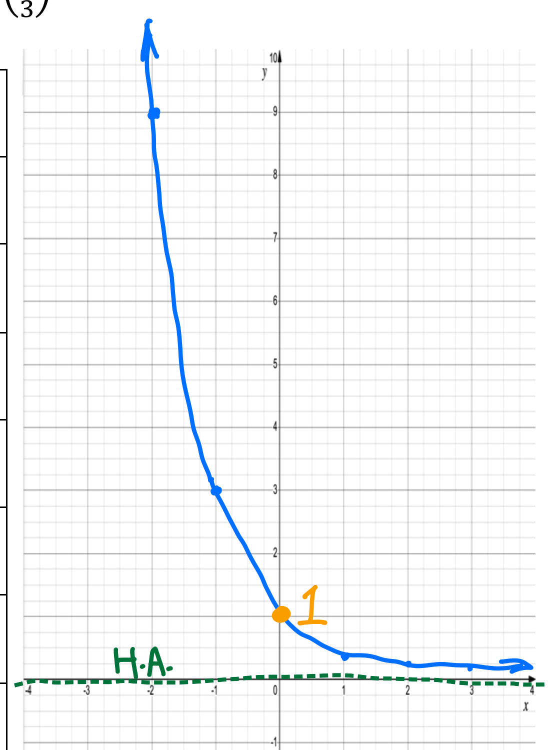


Domain: $(-\infty, \infty)$; Range: $(0, \infty)$.

Asymptote: H.A. $y=0$.

(b) $g(x) = \left(\frac{1}{3}\right)^x$.

x	$g(x)$
-3	27
-2	9
-1	3
0	1
1	$\frac{1}{3}$
2	$\frac{1}{9}$
3	$\frac{1}{27}$



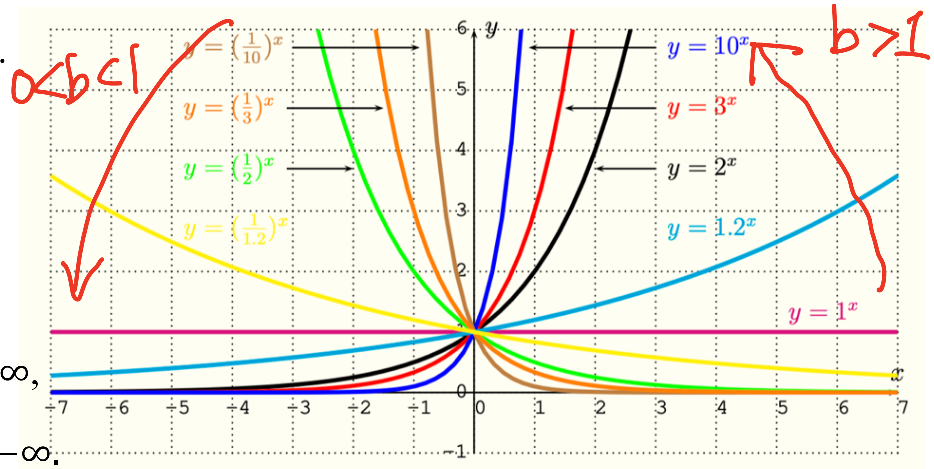
Domain: $(-\infty, \infty)$; Range: $(0, \infty)$.

Asymptote: H.A. $y=0$.

5. Characteristics of Exponential Function of $f(x) = b^x$.

(a) The domain of f: $(-\infty, \infty)$; The Range of f: $(0, \infty)$.

(b) There is No x-intercept. In fact, f approaches, but never touches x-axis which is a horizontal asymptote of f.



(c) Its y-intercept is $(0, 1)$.

(d) f is one-to-one and has an inverse function.

(e) For $b > 1$, $f(x) \to \infty$ as $x \to \infty$,

$f(x) \to 0^+$ as $x \to -\infty$.

(f) For $0 < b < 1$, $f(x) \to 0^+$ as $x \to \infty$, $f(x) \to \infty$ as $x \to -\infty$.

6. What is the 4-steps strategy to find the inverse of a given function? Can it be used to find the inverse function of $f(x) = b^x$?

7. Definition of **Logarithmic Function**:

For $x > 0$ and $b > 0, b \neq 1$, the logarithmic of x with base b is defined by the equivalence

$$\boxed{x = b^y \quad \Leftrightarrow \quad y = \log_b(x).}$$

This computes the inverse of the exponential function $y = b^x$ with base b. (We exchange x and y to get $x = b^y$ and solve for y).

8. Rewrite the equation as a logarithmic equation.

a) $3^4 = x$. $= 81$ $\Updownarrow$ $4 = \log_3 x$, $4 = \log_3(81)$

b) $e^x = 17$. $\Updownarrow$ $x = \log_e 17$ (or $x = \ln 17$)

c) $2^{7a} = 53$. $\Updownarrow$ $7a = \log_2 (53)$

d) $b^3 = 8$. $3 = \log_b (8)$ (in fact, $b = 2$)

9. Rewrite the equation in its equivalent exponential form.

a) $x = \log_2(16)$. $\Updownarrow$ $2^x = 16$ (in fact, $x = 4$)

b) $2 = \log_5 x$. $5^2 = x$ ($x = 25$)

c) $x = \log_{13}(1)$. $13^x = 1$ ($x = 0$)

d) $x = \ln(e^7)$ $= \log_e(e^7)$ $e^x = e^7$ ($x = 7$)