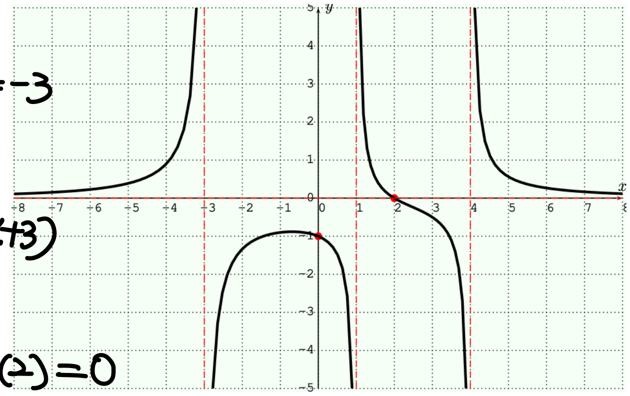
MAT 1375, Classwork10, Fall2024

ID:______________________________ Name:________________________________

1. The graph of $f(x) = \frac{p(x)}{q(x)}$ is displayed below, where deg($p(x)$) =1 and deg($q(x)$)=3.

Find the intercepts, asymptotes, and a formula for $f(x)$.



① x-intercept: $(2, 0)$
② y-intercept: $(0, -1)$
③ V.A: $x=4$, $x=1$, $x=-3$
④ H.A: $y=0$

By ③, $x=4$, $x=1$, $x=-3$ are zeros of $q(x)$
$\Rightarrow q(x) = (x-4)(x-1)(x+3)$

By ①, we know $f(\underset{\text{input}}{2}) = \underset{\text{output}}{0}$, then $\frac{p(2)}{q(2)} = 0 \Rightarrow p(2) = 0$

since deg$(p) = 1$, $p(x) = c \cdot (x-2)$ where c is a constant.

Now $f(x) = \frac{p(x)}{q(x)} = \frac{c(x-2)}{(x-4)(x-1)(x+3)}$, we can use ② ($f(0)=-1$) to find c:

$-1 = f(0) = \frac{c\cdot(0-2)}{(0-4)\cdot(0-1)(0+3)} = \frac{-2\cdot c}{(-4)(-1)(3)} = \frac{-2}{12}c \Rightarrow -1 = \frac{-2}{12}c \Rightarrow c = 6$

Thus $\boxed{f(x) = \frac{6(x-2)}{(x-4)(x-1)(x+3)}}$

For 2. And 3., let $f(x) = \frac{p(x)}{q(x)}$ be a rational function and **deg($p(x)$) > deg($q(x)$)**.

2. Rational Function and Long Division:

If $p(x)$ divided by $q(x)$ can be represented with a quotient $g(x)$ and a remainder $r(x)$

where deg($r(x)$) $\underline{<}$ deg($q(x)$), one can rewrite $f(x)$ as

$$f(x) = \frac{p(x)}{q(x)} = \underline{g(x)} + \frac{r(x)}{q(x)}.$$

3. Asymptotic Behavior with Slant Asymptote:

Since deg($r(x)$) < deg($q(x)$), for large $|x|$ (which is $x \to \pm \underline{\infty}$), we have

$\frac{r(x)}{q(x)}$ approaches $\underline{\text{zero}}$ so that $f(x) \underline{\approx} g(x)$.

If $g(x)$ is a linear function (which is a polynomial of degree $\underline{1}$), then g is called the

$\underline{\text{slant}}$ asymptote of f.

4. Find the **slant asymptote** of the rational function $f(x) = \frac{2x^3-13x^2+35x-26}{x^2-4x+6}$.

Let $p(x) = 2x^3-13x^2+35x-26$, $q(x) = x^2-4x+6$, $f(x) = \frac{p(x)}{q(x)}$

$p(x) = q(x)\cdot g(x) + r(x) = (x^2-4x+6)\cdot(2x-5) + (3x+4)$

$$\begin{array}{r|l} & 2x-5 \\ x^2-4x+6 & 2x^3-13x^2+35x-26 \\ & -(2x^3-8x^2+12x) \\ & -5x^2+23x-26 \\ & -(-5x^2+20x-30) \\ & 3x+4 \end{array}$$

$\Rightarrow f(x) = \frac{p(x)}{q(x)} = g(x) + \frac{r(x)}{q(x)}$

$= 2x-5 + \frac{3x+4}{x^2-4x+6}$

Thus, $y = 2x-5$ is the slant asymptote of f.

5. The Strategy for Solving Inequalities (Application of **Number Line Test**):

Step1. Replace ``>” (``≥”) or ``<” (``≤”) by ``=” and solve the equation.

Step2. Mark the solutions on the number line and check positivity/negativity in each subinterval.

Step3. Check the end points of the subintervals to see if they are included in the solution set.

6. Given $x^3 + 15x \geq 7x^2 + 9$. Solve for x.

Move all the terms to left hand side (LHS): $x^3-7x^2+15x-9 \geq 0$

Step1 change "≥" to "=": and solve it

$x^3-7x^2+15x-9 = 0$ ($x=1$ is a root: $1^3-7\cdot1^2+15\cdot1-9=0$)

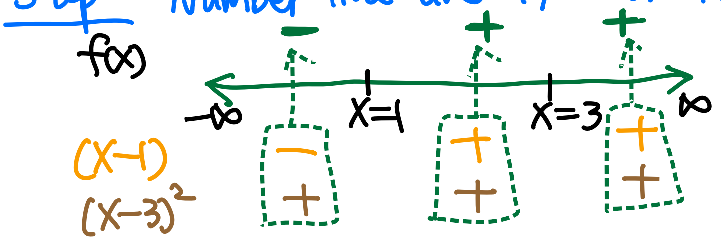
$$\begin{array}{c|cccc} 1 & 1 & -7 & 15 & -9 \\ & & 1 & -6 & -9 \\ \hline & 1 & -6 & 9 & \llcorner 0 \end{array}$$

$\Rightarrow (x-1)(x^2-6x+9) = 0$

$\Rightarrow (x-1)(x-3)^2 = 0$

$\Rightarrow x-1=0$ or $(x-3)=0$ $\Rightarrow x=1$ or 3 (repeat twice)

Step2 Number line are +/− checking:



$x^3-7x^2+15x-9 \geq 0$ we want all the "+" part:

$\Rightarrow$ ① $x \in (1,3)$ and $x \in (3,\infty)$

Step3 Check end points: $x=1$, $x=3$ to see if they satisfy $x^3-7x^2+15x-9 \geq 0$

② for $x=1$, $1^3-7\cdot1^2+15\cdot1-9=0$ ✓ (included)

③ for $x=3$, $3^3-7\cdot3^2-15\cdot3-9=0$ ✓ (included)

By ①, ② & ③, $\boxed{x \in [1,\infty)}$