Mat 1275   HW9

9.4   Exercises

1. Solve $4x - 2 = 0$.

① "+2" on both sides:

② Divided by "4" on both sides

$$4x - 2 = 0$$
$$ +2 \quad +2$$
$$\frac{4x}{4} \quad = \frac{2}{4} \implies x = \frac{2}{4} = \frac{1}{2} \implies \boxed{x = \frac{1}{2}}$$

2. Solve $2(x - 3) = 2x - 3(x - 1)$

① Simplify left hand side (LHS) and Right hand side (RHS)

② isolate "x" on LHS

③ isolate "non-x" on RHS

④ Divided by "3" on both sides

$$2(x - 3) = 2x - 3(x-1)$$
$$\implies 2x - 6 = 2x - 3x + 3$$
$$\implies 2x - 6 = -x + 3$$
$$ +x \qquad\quad +x$$
$$3x - 6 \quad = \quad 3$$
$$ +6 \qquad\qquad +6$$
$$\frac{3x}{3} \qquad = \qquad \frac{9}{3}$$
$$\implies \boxed{x = 3}$$

3. Solve $(x - 2)(7x - 3) = 0$.

① $\square \cdot \triangle = $ zero

$\implies \square = $ zero or $\triangle = $ zero

② isolate "x"

①$\implies$ $x - 2 = 0$ or $7x - 3 = 0$
$ +2 \ +2 \qquad\qquad +3 \ +3$

②$\implies$ $x = 2$ or $\frac{7x}{7} = \frac{3}{7}$

$\implies \boxed{x = 2 \quad \text{or} \quad x = \frac{3}{7}}$

4. Solve $10x^2 - 9x + 2 = 0$.

① Factor LHS (left hand side)

② $\square \cdot \triangle = $ zero

$\implies \square = $ zero or $\triangle = $ zero

③ isolate "x"

$$10x^2 - 9x + 2 = 0$$

2x ⤫ -1
5x   -2

$-4x - 5x = -9x$

$\implies (2x - 1)(5x - 2) = 0$

$\implies 2x - 1 = 0$ or $5x - 2 = 0$
$ +1 \ +1 \qquad\qquad +2 \ +2$

$\implies \frac{2x}{2} = \frac{1}{2}$ or $\frac{5x}{5} = \frac{2}{5}$

$\implies \boxed{x = \frac{1}{2} \quad \text{or} \quad x = \frac{2}{5}}$

5. Solve $x(3x - 7) = -2$.

① Simplify LHS:

$$x(3x-7) = 2$$
$$\Rightarrow 3x^2 - 7x = 2$$
$$\qquad \qquad \underline{-2 \quad -2}$$
$$\Rightarrow 3x^2 - 7x - 2 = 0$$

② Factor LHS

$$\begin{array}{ccc} x & & -2 \\ 3x & \times & -1 \end{array}$$

$$-x - 6x = -7x$$

$$\Rightarrow (x-2)(3x-1) = 0$$

③ $\square \cdot \triangle = \text{zero}$

$\Rightarrow \square = \text{zero or } \triangle = \text{zero}$

$$\Rightarrow \quad x - 2 = 0 \quad \text{or} \quad 3x - 1 = 0$$
$$\qquad \quad \underline{+2 \ +2} \qquad \qquad \underline{+1 \ +1}$$

④ isolate "x"

$$\Rightarrow \quad x = 2 \quad \text{or} \quad \frac{3x}{3} = \frac{1}{3}$$

$$\Rightarrow \quad \boxed{x = 2 \quad \text{or} \quad x = \frac{1}{3}}$$

6. Suppose that you have a poster which is 2 feet by 3 feet. Suppose you can enlarge the poster by including a white border as long as its area doesn't exceed $8\frac{3}{4}$ square feet. What is the width of the largest uniform border you could have? Include a labeled picture and an appropriate equation.
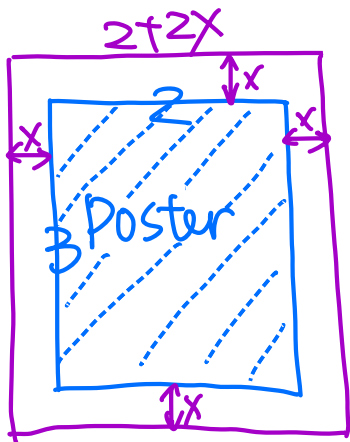
Let the width of the white border be x feet.
The post with white border is $2+2x$ by $3+2x$ and its area is $8\frac{3}{4} = \frac{35}{4}$

$$(2+2x)(3+2x) = \frac{35}{4}$$

① times "4" on both sides:

$$\Rightarrow 4x^2 + 10x + 6 = \frac{35}{4}$$
$$\Rightarrow 4(4x^2 + 10x + 6) = \cancel{4} \cdot \frac{35}{\cancel{4}}$$
$$\Rightarrow 16x^2 + 40x + 24 = 35$$
$$\qquad \qquad \qquad \underline{-35 \quad -35}$$
$$\Rightarrow 16x^2 + 40x - 11 = 0$$

$$\begin{array}{ccc} 4x & & -1 \\ 4x & \times & +11 \end{array}$$

$$44x - 4x = 40x$$

② $\square \cdot \triangle = \text{zero}$

$\Rightarrow \square = \text{zero or } \triangle = \text{zero}$

③ isolate "x"

$$\Rightarrow (4x-1) \cdot (4x+11) = 0$$
$$\Rightarrow 4x - 1 = 0 \quad \text{or} \quad 4x + 11 = 0$$
$$\qquad \quad \underline{+1 \ +1} \qquad \qquad \underline{-11 \ -11}$$
$$\Rightarrow \quad 4x = 1 \qquad \text{or} \quad 4x = -11$$
$$\Rightarrow \quad \boxed{x = \frac{1}{4}} \qquad \text{or} \quad x = -\frac{11}{4} \quad \text{(the answer has to be "+")}$$

7. Suppose an object has a height $-16t^2 + 78t + 10$ at when the stopwatch reads t seconds. At what time does it hit the ground? At what time is 10 feet above the ground?

Information from Question: $\text{height} = -16t^2 + 78t + 10$ where t is time.

① Hit the ground $\Rightarrow$ height $= 0$

$-16t^2 + 78t + 10 = 0$

$\Rightarrow -2(8t^2 - 39t - 5) = 0$

GCF

$\Rightarrow \dfrac{-2(8t^2 - 39t - 5)}{-2} = \dfrac{0}{-2}$ (divided by "-2" on both sides)

$\Rightarrow 8t^2 - 39t - 5 = 0$ (factor it)

$\quad t \qquad -5$
$\quad 8t \qquad 1$

$\Rightarrow (t-5) \cdot (8t+1) = 0$ $\Big(\square \cdot \triangle = \text{zero}$
$\Rightarrow t-5 = 0 \text{ or } 8t+1 = 0$ $\Rightarrow \square = \text{zero or}$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad \triangle = \text{zero}\Big)$

$\Rightarrow t - 5 = 0, \quad 8t + 1 = 0$
$\quad\quad +5 \ +5 \quad\quad\quad -1 \ \ -1$
$\quad\quad t = 5 \quad\quad\quad 8t = -1$
$\quad\quad\quad\quad\quad\quad \Rightarrow t = -\frac{1}{8}$

$\Rightarrow t = 5$ (since $t = -\frac{1}{8} < 0$ is not an answer)

② What time is 10 feet above ground?
$\Rightarrow$ height $= 10$

$-16t^2 + 78t + 10 = 10$
$\quad\quad\quad\quad\quad\quad -10 \ \ -10$
$\overline{-16t^2 + 78t \quad\quad = 0}$

$\Rightarrow -2t \cdot (8t - 39) = 0$

$\Rightarrow \dfrac{-2t}{-2} = \dfrac{0}{-2} \text{ or } 8t - 39 = 0$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad +39 \ +39$

$\Rightarrow t = 0 \text{ or } \dfrac{8t}{8} = \dfrac{39}{8}$

$\Rightarrow \boxed{t = 0 \text{ or } t = \dfrac{39}{8}}$ both are the answers