

Mat 1275 HW8

8.4 Exercises

1. Evaluate exactly and estimate without a calculator at 4: $\sqrt[3]{10x^2}$.

Sol: Keywords: Evaluate, estimate, $x=4$, $\sqrt[3]{10x^2}$ · cubic root of $10x^2$

$$\Rightarrow \sqrt[3]{10(4)^2} = \sqrt[3]{5 \cdot 2 \cdot 4^2} = \sqrt[3]{5 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \sqrt[3]{5 \cdot 2 \cdot 2 \cdot 2^3} = \sqrt[3]{20} \cdot \sqrt[3]{2^3} = \sqrt[3]{20} \cdot 2 = 2 \sqrt[3]{20}$$

2. Simplify $3\sqrt[4]{48x^9y^{18}z^6}$.

Sol: Keyword: Simplify, $3 \cdot \sqrt[4]{48x^9y^{18}z^6}$
 multiply (pointing to 3), fourth root (pointing to the radical symbol)

① $48 = 6 \times 8$

$= 3 \times 2 \times 2 \times 2 \times 2 = 3 \times 2^4$

② $\sqrt[4]{x^4} = x$, $x^9 = x^4 \cdot x^4 \cdot x$

③ $\sqrt[4]{y^4} = y$, $y^{18} = y^4 \cdot y^4 \cdot y^4 \cdot y^4 \cdot y^2$

④ $\sqrt[4]{z^4} = z$, $z^6 = z^4 \cdot z^2$

$\Rightarrow 3 \sqrt[4]{48x^9y^{18}z^6}$

$= 3 \sqrt[4]{3 \cdot 2^4 \cdot x^4 \cdot x^4 \cdot x \cdot y^4 \cdot y^4 \cdot y^4 \cdot y^4 \cdot y^2 \cdot z^4 \cdot z^2}$

$= 3 \sqrt[4]{3 \sqrt[4]{2^4} \cdot \sqrt[4]{x^4} \cdot \sqrt[4]{x^4} \cdot \sqrt[4]{x} \cdot \sqrt[4]{y^4} \cdot \sqrt[4]{y^4} \cdot \sqrt[4]{y^4} \cdot \sqrt[4]{y^4} \cdot \sqrt[4]{y^2} \cdot \sqrt[4]{z^4} \cdot \sqrt[4]{z^2}}$

$= 3 \cdot 2 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot z \cdot \sqrt[4]{3 \sqrt[4]{x} \sqrt[4]{y^2} \sqrt[4]{z^2}}$

$= 6x^2y^4z \sqrt[4]{3xy^2z^2}$

3. Evaluate $8^{2/3}$.

Sol keyword: Evaluate, fraction exponent $\frac{2}{3}$

① $8^{a/b} = (8^a)^{1/b}$

② $(\cdot)^{1/b} = \sqrt[b]{\cdot}$
 a number (pointing to the dot)

③ $\sqrt[3]{2^3} = 2$

$8^{2/3} \stackrel{①}{=} (8^2)^{1/3} \stackrel{②}{=} \sqrt[3]{8^2} \stackrel{③}{=} \sqrt[3]{2^3 \cdot 2^3}$

$8^2 = (2^3)^2 = 2^3 \cdot 2^3$

$= \sqrt[3]{2^3} \sqrt[3]{2^3} \stackrel{③}{=} 2 \cdot 2 = 4$

4. Simplify $\left(2\sqrt{\frac{2x^4y^3}{45z^7}}\right)^3$.

Sol Keyword: simplify,

① $(a \cdot b)^3 = a^3 b^3$

② $(\sqrt{a})^3 = a^{\frac{3}{2}} = (a^3)^{\frac{1}{2}} = \sqrt{a^3}$

③ $(a^4)^3 = a^{4 \cdot 3} = a^{12}$

$(b^3)^3 = b^{3 \cdot 3} = b^9$

$(c^7)^3 = c^{7 \cdot 3} = c^{21}$

$$\left(2\sqrt{\frac{2x^4y^3}{45z^7}}\right)^3$$

$$\stackrel{\textcircled{1}}{=} 2^3 \left(\sqrt{\frac{2x^4y^3}{45z^7}}\right)^3 \stackrel{\textcircled{2}}{=} 8 \sqrt{\left(\frac{2x^4y^3}{45z^7}\right)^3}$$

$$\stackrel{\textcircled{3}}{=} 8 \sqrt{\frac{2^3 x^{12} y^9}{45^3 z^{21}}} = 8 \sqrt{\frac{z^2 \cdot z \cdot x^{12} \cdot y^8 y}{45^2 \cdot 45 \cdot z^{20} z}}$$

$$= 8 \sqrt{\frac{z^2 x^{12} y^8}{45^2 \cdot z^{20}}} \cdot \sqrt{\frac{2y}{45z}}$$

$$= 8 \cdot \frac{z \cdot x^6 y^4}{45 \cdot z^{10}} \cdot \sqrt{\frac{2y}{45z}} = \frac{16x^6y^4}{45z^{10}} \sqrt{\frac{2y}{45z}}$$

5. Simplify $5\sqrt{20} - 3\sqrt{45}$.

Sol Keyword: simplify

① $20 = 4 \times 5 = 2^2 \cdot 5$

② $45 = 9 \times 5 = 3^2 \cdot 5$

$$5\sqrt{20} - 3\sqrt{45}$$

$$\stackrel{\textcircled{1}, \textcircled{2}}{=} 5 \cdot \sqrt{2^2 \cdot 5} - 3 \sqrt{3^2 \cdot 5}$$

$$= 5 \cdot 2 \cdot \sqrt{5} - 3 \cdot 3 \cdot \sqrt{5} = 10\sqrt{5} - 9\sqrt{5} = \sqrt{5}$$

6. Multiply and simplify $(7\sqrt{3} + 2\sqrt{5})(2\sqrt{3} - 3\sqrt{5})$.

Sol Keyword: Multiply, simplify

$2 \cdot 7 \cdot 3 = 42$

$7\sqrt{3}$	$+ 2\sqrt{5}$
$2\sqrt{3}$	$\begin{matrix} 2 \cdot 7 \cdot 3 \\ 2 \cdot 2 \cdot \sqrt{15} \end{matrix}$
$-3\sqrt{5}$	$\begin{matrix} -3 \cdot 7 \cdot \sqrt{15} \\ -3 \cdot 2 \cdot 5 \end{matrix}$

$4\sqrt{15}$

$-21\sqrt{15}$

$-3 \cdot 2 \cdot 5 = -30$

$$(7\sqrt{3} + 2\sqrt{5})(2\sqrt{3} - 3\sqrt{5})$$

$$= 42 - 30 + 4\sqrt{15} - 21\sqrt{15} = 12 - 17\sqrt{15}$$

7. Multiply and simplify $(\sqrt{3} + 4\sqrt{5})(\sqrt{3} - 4\sqrt{5})$.

Sol. Keyword: Multiply, simplify, conjugate

	$\sqrt{3}$	$+4\sqrt{5}$
$\sqrt{3}$	3	$4\sqrt{5}$
$-4\sqrt{5}$	$-4\sqrt{5}$	$-4 \cdot 4 \cdot 5$

$-4 \cdot 4 \cdot 5 = -80$

$$(\sqrt{3} + 4\sqrt{5})(\sqrt{3} - 4\sqrt{5}) = 3 - 80 + 4\sqrt{5} - 4\sqrt{5} = 3 - 80 = \boxed{-77}$$

8. Divide and simplify $\frac{7\sqrt{3} + 2\sqrt{5}}{2\sqrt{3} - 3\sqrt{5}}$.

Sol. Keyword: Divide, simplify, Rationalize, conjugate.

① Rationalize with conjugate \rightarrow

$$(2\sqrt{3} - 3\sqrt{5})(2\sqrt{3} + 3\sqrt{5})$$

	$2\sqrt{3}$	$-3\sqrt{5}$
$2\sqrt{3}$	$2 \cdot 2 \cdot 3 = 12$	$-3 \cdot 2 \cdot \sqrt{15} = -6\sqrt{15}$
$3\sqrt{5}$	$2 \cdot 3 \cdot \sqrt{15} = 6\sqrt{15}$	$-3 \cdot 3 \cdot 5 = -45$

$$\frac{7\sqrt{3} + 2\sqrt{5}}{2\sqrt{3} - 3\sqrt{5}} \cdot \frac{(2\sqrt{3} + 3\sqrt{5})}{(2\sqrt{3} + 3\sqrt{5})} = \frac{(7\sqrt{3} + 2\sqrt{5})(2\sqrt{3} + 3\sqrt{5})}{(2\sqrt{3} - 3\sqrt{5})(2\sqrt{3} + 3\sqrt{5})}$$

$$\stackrel{2 \cdot 7 \cdot 3 = 42}{=} \frac{42 + 30 + 4\sqrt{15} + 21\sqrt{15}}{-33}$$

$$= \frac{72 + 25\sqrt{15}}{-33}$$

	$2\sqrt{3}$	$3\sqrt{5}$
$2\sqrt{3}$	$2 \cdot 2 \cdot 3 = 12$	$2 \cdot 2 \cdot \sqrt{15} = 4\sqrt{15}$
$3\sqrt{5}$	$3 \cdot 2 \cdot \sqrt{15} = 6\sqrt{15}$	$3 \cdot 3 \cdot 5 = 45$

9. Simplify and write your answer using radical notation (assume x and y are positive):

Sol. Keyword: Simplify, radical notation

$$\begin{aligned} \textcircled{1} \sqrt[3]{a} &= (a^{\frac{1}{2}})^{\frac{1}{3}} \\ &= a^{\frac{1}{2} \cdot \frac{1}{3}} = a^{\frac{1}{6}} = \sqrt[6]{a} \end{aligned}$$

$$\left(\frac{3\sqrt{xy^3}}{4\sqrt[3]{(9x)^2\sqrt{3y}}} \right)^2 = \left(\frac{3y\sqrt{xy}}{4\sqrt[3]{(9x)^2} \cdot \sqrt[3]{3y}} \right)^2$$

$$= \frac{3^2 y^2 (\sqrt{xy})^2}{4^2 (\sqrt[3]{(9x)^2})^2 (\sqrt[3]{3y})^2} = \frac{3^2 y^2 xy}{4^2 \sqrt[3]{(9x)^4} \sqrt[3]{(3y)^2}}$$

$$= \frac{3^2 xy^3}{4^2 \cdot 9 \sqrt[3]{9x} \cdot \sqrt[3]{3y}} = \frac{y^3}{16 \sqrt[3]{3 \cdot 3 \cdot 3 \cdot xy}} = \frac{y^3}{16 \cdot 3 \cdot \sqrt[3]{xy}}$$

$$= \frac{y^3}{48 \sqrt[3]{xy}}$$

