

6.4 Exercises

- Factor the GCF of $15x^5y^2 - 20x^3y^4z + 5x^2y^2$ out. Check your answer by distributing.
- Factor $27x^4 - 18x^3 - 24x^2$ completely. Check your answer by distributing.
- Factor $4y^2 - 9x^2$ completely. Check your answer by distributing.

Sol:

1. To find the GCF (Greatest Common Factor) of $15x^5y^2 - 20x^3y^4z + 5x^2y^2$,

We will split these three terms into 4 parts:

$$15x^5y^2 \cdot 1 - 20x^3y^4z + 5x^2y^2 \cdot 1$$

Coefficient

15	20	5
1x15	1x20	1x5
3x5	2x10	↓
↓	4x5	↓
↓	↓	↓
1, 3, 5, 15	1, 2, 4, 5, 10, 20	1, 5

Common divisor: 1, 5

Greatest Common divisor: 5

x-part

$$x^5, x^3, x^2$$

Greatest Common factor of x part: x^2

y-part

$$y^2, y^4, y^2$$

GCF of y parts: y^2

z-part

$$1, z, 1$$

GCF of z part: 1

$$5x^2y^2 \cdot 1 (3x^3 \cdot 1 \cdot 1 - 4x \cdot y^2z + 1 \cdot 1 \cdot 1)$$

$$= 5x^2y^2 (3x^3 - 4xy^2z + 1)$$

2. To factor $27x^4 - 18x^3 - 24x^2$ completely, based on observation, we can take out GCF from it to simplify the process:

$$27x^4 - 18x^3 - 24x^2 = 3x^2(9x^2 - 6x - 8)$$

Now, we can focus on $9x^2 - 6x - 8$ first:

I split $9x^2$ into two parts:

① $9x^2 = 3x \cdot 3x$
 ② $9x^2 = x \cdot 9x$) two possibilities

II split -8 into two numbers:

① $-8 = (-1) \times 8$	⑤ $-8 = 8 \times (-1)$	} 8 possibilities
② $-8 = (-2) \times 4$	⑥ $-8 = 4 \times (-2)$	
③ $-8 = (-4) \times 2$	⑦ $-8 = 2 \times (-4)$	
④ $-8 = (-8) \times 1$	⑧ $-8 = 1 \times (-8)$	

* For I ①, we only need to check ①, ②, ③, ④ in part II

For I ②, we need to check all 8 possibilities.

III Test the possibilities by doing the products crossly, summing the products up, and check if the sum matches the middle term:

①

$$\begin{array}{l} 3x \nearrow (-1) \\ 3x \searrow 8 \end{array}$$

$$8 \cdot 3x + (-1) \cdot 3x = 24x - 3x = 21x \Rightarrow \text{not matching}$$

②

$$\begin{array}{l} 3x \nearrow (-2) \\ 3x \searrow 4 \end{array}$$

$$4 \cdot 3x + (-2) \cdot 3x = 12x - 6x = 6x \Rightarrow \text{not matching but it only misses a "-" from the middle term}$$

switching " - " \rightarrow ⑦

⑦

$$\begin{array}{l} 3x \nearrow 2 \\ 3x \searrow (-4) \end{array}$$

$$(-4) \cdot 3x + 2 \cdot 3x = -12x + 6x = -6x \Rightarrow \text{matching!}$$

IV Write down the result:

$$27x^4 - 18x^3 - 24x^2 = 3x^2(9x^2 - 6x - 8) = \boxed{3x^2(3x + 2)(3x - 4)}$$

$\begin{array}{c} 3x \\ 3x \end{array}$
 $\begin{array}{c} 2 \\ (-4) \end{array}$

3. To factor $4y^2 - 9x^2$, we observe

① $4y^2 = (2y)^2$ and $9x^2 = (3x)^2$

② Using $a^2 - b^2 = (a+b)(a-b)$, we have

$$4y^2 - 9x^2 = (2y)^2 - (3x)^2 = (2y + 3x)(2y - 3x).$$