

# Mat 1275 HW12

## 12.4 Exercises

1. Rewrite  $x^3 + 5x^2 + 8x + 4 = 0$  so that the polynomial on the left is in factored form. Use this form to solve the equation.

Sol  $x^3 + 5x^2 + 8x + 4 = 0$

$$\Rightarrow \frac{x^3 + x^2 + 4x^2 + 4x + 4x + 4}{\downarrow \quad \downarrow \quad \downarrow} = 0$$

$$\Rightarrow \frac{x^2(x+1) + 4x(x+1) + 4(x+1)}{\downarrow \quad \downarrow \quad \downarrow} = 0$$

$$\Rightarrow (x+1)(\cancel{x^2} + 4x + 4) = 0$$

$\begin{array}{ccc} x & & +2 \\ x & \times & +2 \\ \hline & & 2x+2x=4x \end{array}$

$$\Rightarrow (x+1)(x+2)(x+2) = 0$$

$$\Rightarrow \begin{array}{ccc} x+1=0 & \text{or} & x+2=0 & \text{or} & x+2=0 \\ -1 & -1 & -2 & -2 & -2 & -2 \end{array}$$

$$\Rightarrow x = -1 \text{ or } x = -2 \text{ or } x = -2$$

2. Solve  $(3x + 2)(2x^2 - x + 2) = 0$ .

Sol:  $(3x+2)(2x^2-x+2) = 0$

$$\Rightarrow \begin{array}{l} 3x+2=0 \\ -2 \quad -2 \end{array} \text{ or } 2x^2 - 1x + 2 = 0$$

$A=2, B=-1, C=2$

$$\Rightarrow \frac{3x}{3} = \frac{-2}{3}$$

$$\Rightarrow x = -\frac{2}{3}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 2 \cdot 2}}{2 \cdot 2}$$

$1 - 16 = -15$   
 $\sqrt{-15} = \sqrt{15} \cdot \sqrt{-1}$   
 $= \sqrt{15}i$

$$= \frac{1 \pm \sqrt{15}i}{4}$$

$$\Rightarrow x = -\frac{2}{3}, \quad x = \frac{1 + \sqrt{15}i}{4}, \quad x = \frac{1 - \sqrt{15}i}{4}$$

3. Solve  $x^3 - 5x^2 + 8x - 4 = 0$  using the fact that 1 is a zero of  $x^3 - 5x^2 + 8x - 4$ .

Sol: Keyword: 1 is a zero of  $x^3 - 5x^2 + 8x - 4$   
 $\Rightarrow (x-1)$  is a factor of  $x^3 - 5x^2 + 8x - 4$

$$x^3 - 5x^2 + 8x - 4 = 0$$

$$\Rightarrow \frac{x^3 - x^2 - 4x^2 + 4x + 4x - 4}{\downarrow \quad \downarrow \quad \downarrow} = 0$$

$$\Rightarrow \frac{x^2(x-1) - 4x(x-1) + 4(x-1)}{\downarrow \quad \downarrow \quad \downarrow} = 0$$

$$\Rightarrow (x-1)(x^2 - 4x + 4) = 0 \Rightarrow (x-1)(x-2)(x-2) = 0$$

$$\begin{array}{r} x & & -2 \\ x & \times & -2 \\ \hline & -2x & -2x \\ & & -4x \end{array}$$

$$\Rightarrow (x-1) = 0 \text{ or } (x-2) = 0 \text{ or } (x-2) = 0$$

$$\begin{array}{ccc} +1 & +1 & \\ +2 & +2 & \\ +2 & +2 & \end{array}$$

$$\Rightarrow x=1 \text{ or } x=2 \text{ or } x=2.$$

4. Solve  $x^3 - 2x^2 - 5x - 2 = 0$  (hint:  $-1$  is one solution).

Sol: Keyword:  $-1$  is one solution  $\Rightarrow (x+1)$  is a factor of  $x^3 - 2x^2 - 5x - 2 = 0$ .

$$\Rightarrow x^3 - 2x^2 - 5x - 2 = 0$$

$$\Rightarrow \begin{array}{r} x^3 + x^2 - 3x^2 - 3x - 2x - 2 = 0 \\ \hline \downarrow \quad \quad \downarrow \quad \quad \downarrow \end{array}$$

$$\Rightarrow x^2(x+1) - 3x(x+1) - 2(x+1) = 0$$

$$\Rightarrow (x+1)(x^2 - 3x - 2) = 0$$

$$\Rightarrow x+1 = 0 \text{ or } 1x^2 - 3x - 2 = 0$$

$$\begin{array}{ccc} +1 & -1 & \\ A=1, & B=-3, & C=-2 \end{array}$$

$$\Rightarrow x = -1 \quad x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$9 + 8 = 17$

$$\Rightarrow x = -1 \text{ or } x = \frac{3 \pm \sqrt{17}}{2}$$

$$\Rightarrow x = -1 \text{ or } x = \frac{3 + \sqrt{17}}{2} \text{ or } x = \frac{3 - \sqrt{17}}{2}$$