Mat 1275 HW 10

## 10.4 Exercises

1. Solve $x^2 = 20$.

Sol: Key word: "Solve" means "solve for $x$",

"=" means this is an equation, not an expression

since it is $x$ "square", we're looking for two answers.

$x^2 = 20 = (\pm\sqrt{20})^2$

① $20 = (\pm\sqrt{20})^2$

② Use "$\sqrt{\ }$" to cancel "the square"

$\Rightarrow \sqrt{x^2} = \sqrt{(\pm\sqrt{20})^2} \Rightarrow x = \pm\sqrt{20}$

$\Rightarrow x = +\sqrt{20}, \ x = -\sqrt{20}$

③ $20 \begin{smallmatrix} 5 \\ 4 \end{smallmatrix} \Rightarrow 20 = 4 \cdot 5$

$\Rightarrow \boxed{x = +2\sqrt{5}, \ x = -2\sqrt{5}}$

$\Rightarrow \sqrt{20} = \sqrt{4} \cdot \sqrt{5} = 2 \cdot \sqrt{5}$

2. Solve $(x-2)^2 = 12$.

Sol: Keyword: "Solve" means "solve for $x$"

"=" means this is an equation, not an expression.

"the square" means we are expecting two answers.

① $12 = (\pm\sqrt{12})^2$

$(x-2)^2 = 12 = (\pm\sqrt{12})^2$

② Use "$\sqrt{\ }$" to "cancel" the square

$\Rightarrow \sqrt{(x-2)^2} = \sqrt{(\pm\sqrt{12})^2} \Rightarrow (x-2) = \pm\sqrt{12}$

$+2 \qquad +2$

$\Rightarrow x = 2 \pm \sqrt{12}$

③ $12 = 4 \cdot 3$

$\Rightarrow x = 2 + \sqrt{12}$ or $x = 2 - \sqrt{12}$

$\Rightarrow \sqrt{12} = \sqrt{4} \cdot \sqrt{3} = 2 \cdot \sqrt{3}$

$\Rightarrow x = 2 + 2\sqrt{3}$ or $x = 2 - 2\sqrt{3}$

3. Solve $x^2 - 4x = 6$.

Sol: Keyword: "Solve" means "solve for $x$",

"=" means it is an equation, not an expression

There are several ways that we could try to solve for $x$.

• factor • complete the square • Quadratic formula

① Move "6" to left hand side by "−6" on both sides

$$x^2 - 4x = 6$$
$$\quad -6 \quad -6$$
$$\Rightarrow x^2 - 4x - 6 = 0$$

② Try factor ⇒ fails, Try Complete the square: or

$$x^2 + \underline{2a}x + a^2 = (x+a)^2$$
$$x^2 \underline{-4}x + \underline{4} = (x-2)^2$$
$$(2a = -4 \Rightarrow a = -2 \Rightarrow a^2 = 4)$$

$$\Rightarrow x^2 - 4x + 4 - 6 = 0 + 4$$
$$\Rightarrow (x-2)^2 - 6 = 4$$
$$\quad +6 \quad +6$$
$$\Rightarrow (x-2)^2 = 10 = (\pm\sqrt{10})^2$$
$$\Rightarrow \sqrt{(x-2)^2} = \sqrt{(\pm\sqrt{10})^2}$$
$$\Rightarrow x - 2 = \pm\sqrt{10}$$
$$\quad +2 \quad +2$$
$$\Rightarrow x = 2 \pm \sqrt{10}$$
$$\Rightarrow \boxed{x = 2 + \sqrt{10} \text{ or } x = 2 - \sqrt{10}.}$$

② Use Quadratic Formula

$$Ax^2 + Bx + C = 0$$
$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$1x^2 - 4x - 6 = 0$$
$$A = 1, B = -4, C = -6$$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot (-6)}}{2 \cdot 1}$$
$$= \frac{4 \pm \sqrt{16 + 24}}{2} \qquad 16+24 = 40$$
$$\boxed{\sqrt{40} = \sqrt{4} \cdot \sqrt{10} = 2 \cdot \sqrt{10}}$$
$$= \frac{4 \pm 2\sqrt{10}}{2} = \frac{\cancel{2}(2 \pm \sqrt{10})}{\cancel{2}}$$
$$= 2 \pm \sqrt{10}.$$

4. Solve $3x^2 - 4x = 20$ using the quadratic formula.

Sol: Keyword: "Solve" means "solve for $x$"

quadratic formula: $Ax^2 + Bx + C = 0 \Rightarrow x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

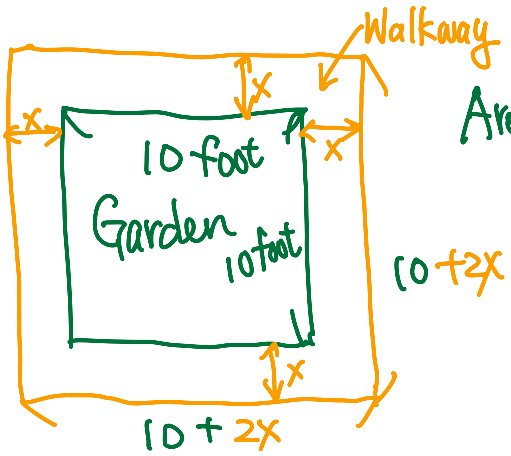
$$3x^2 - 4x = 20$$
$$\quad -20 \quad -20$$
$$\Rightarrow 3x^2 - 4x - 20 = 0$$
$$A = 3, \; B = -4, \; C = -20$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4\cdot 1\cdot(-20)}}{2\cdot 1} = \frac{4 \pm \sqrt{16+80}}{2} \quad 96$$

$$\boxed{\sqrt{96} = \sqrt{16}\cdot\sqrt{6} = 4\sqrt{6}} = \frac{4 \pm 4\sqrt{6}}{2} = \frac{\cancel{2}(2 \pm 2\sqrt{6})}{\cancel{2}} = 2 \pm 2\sqrt{6}$$

5. Suppose you are trying to make a square garden with a walkway of uniform width. You only have enough garden materials for a 10 foot by 10 foot gardening patch. How wide should your walkway be so that the total area (walkway and garden) is 120 square feet?

Sol: Keyword: • 10 x 10 "square" garden;
• Walkway of uniform width ⇒ $x$ foot
• total area = 120 square feet




Area of garden + Area of Walkway = total area

$$(10+2x)^2 = 120 = (\pm\sqrt{120})^2$$

120 — 5, 24 — 6, 4

$$\Rightarrow \sqrt{(10+2x)^2} = \sqrt{(\pm\sqrt{120})^2}$$

$$\Rightarrow 10+2x = \pm\sqrt{120}$$
$$-10 \qquad -10$$

$$\Rightarrow \frac{2x}{2} = \frac{-10 \pm \sqrt{120}}{2}$$

$$\Rightarrow x = \frac{-10 \pm \sqrt{120}}{2} = \frac{-10 \pm 2\sqrt{30}}{2}$$

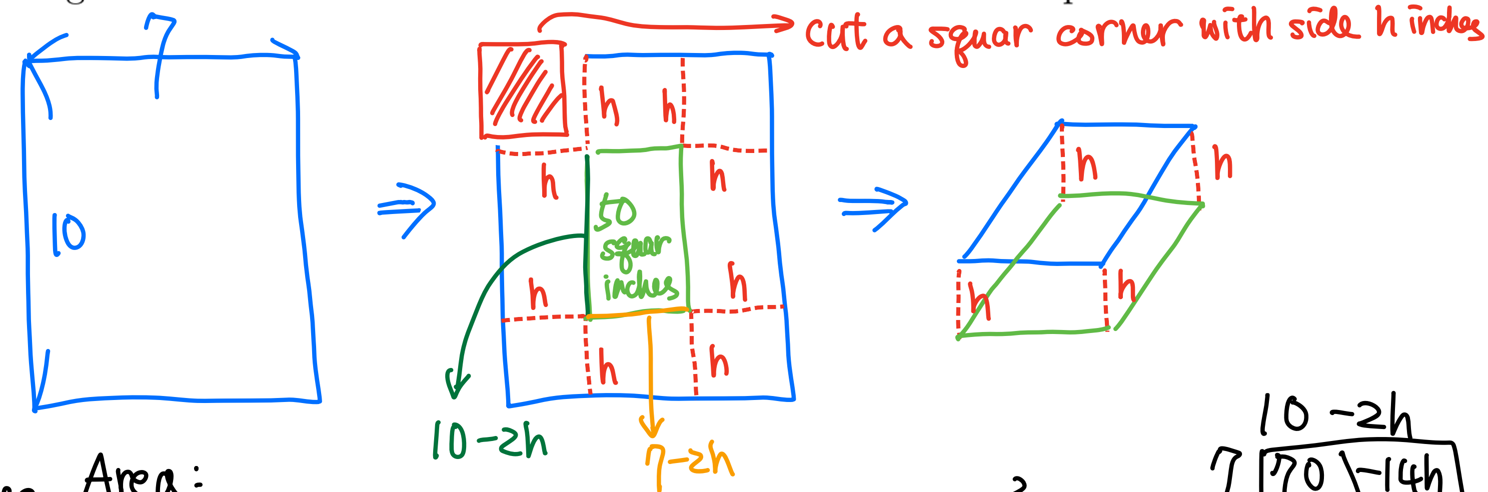
$$\sqrt{120} = \sqrt{4}\cdot\sqrt{30} = 2\sqrt{30}$$

$$\Rightarrow x = \frac{\cancel{2}(-5 \pm \sqrt{30})}{\cancel{2}} = -5 \pm \sqrt{30}$$

But $x$ has to be positive, the $x = -5 + \sqrt{30}$. (feet).

6. Suppose you want to form a box with an open top by cutting out corners of a rectangular piece of cardboard which is 10 inches by 7 inches. How high will the box be if the area of the base of the box is 50 square inches?

Sol:




Base Area:

$$(10-2h)\cdot(7-2h)=50 \Rightarrow 70-14h-20h+4h^2=50$$

$$-50 \qquad -50$$

| | $10$ | $-2h$ |
|---|---|---|
| $7$ | $70$ | $-14h$ |
| $-2h$ | $-20h$ | $4h^2$ |

$$\Rightarrow 4h^2-34h+20=0 \Rightarrow \underbrace{2}_{\text{GCF}}(2h^2-17h+10)=0$$

$$\Rightarrow 2h^2-17h+10=0$$

$A=2$, $B=-17$, $C=10$

$$h=\frac{-(-17)\pm\sqrt{(-17)^2-4\cdot 2\cdot 10}}{2\cdot 2}=\frac{17\pm\sqrt{209}}{4}$$

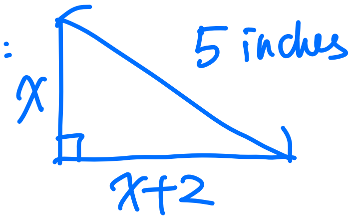
$$17^2-80=289-80=209$$

Since $h=\frac{17+\sqrt{209}}{4}$ makes $7-2h<0$ which is impossible.

Then $\boxed{h=\frac{17-\sqrt{209}}{4}}$

7. Suppose that a right triangle has a hypotenuse of length 5 inches and one of the legs is 2 inches more than the other. What are the lengths of the legs?

Sol:




By Pythagorean theorem, we have

$$5^2 = x^2 + (x+2)^2$$

$$\Rightarrow 25 = x^2 + x^2 + 4x + 4$$

$$\Rightarrow 25 = 2x^2 + 4x + 4$$

$$-25 \qquad\qquad -25$$

$$\Rightarrow 0 = 2x^2 + 4x - 21$$

$$A = 2, \; B = 4, \; C = -21$$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4 \cdot 2 \cdot (-21)}}{2 \cdot 2}$$

$$16 + 168 = 184$$

$$184 \to 8, 23; \quad 8 \to 4, 2$$

$$\sqrt{184} = \sqrt{4}\sqrt{2} \cdot \sqrt{23} = 2 \cdot \sqrt{2} \cdot \sqrt{23} = 2\sqrt{46}$$

$$= \frac{-4 \pm 2\sqrt{46}}{4} = \frac{\cancel{2}(-2 \pm \sqrt{46})}{\cancel{4}\, 2} = \frac{-2 \pm \sqrt{46}}{2}$$

Since leg has a positive length, then $x = \frac{-2 + \sqrt{46}}{2}$

One leg is $\frac{-2+\sqrt{46}}{2}$ inches

and the other is $2 + \frac{-2+\sqrt{46}}{2} = \frac{4 - 2 + \sqrt{46}}{2} = \frac{2+\sqrt{46}}{2}$ inches.