Math 1451, Honor Calculus Practice1, Spring 2016.

January 22, 2016

PSID: ______________________ Name: Sol

1. Given two vectors $\mathbf{u} = \langle b^2 - b, -2b, 2\rangle$ and $\mathbf{v} = \langle b-1, b, b\rangle$. Find the range of b such that the angle between $\mathbf{u}$ and $\mathbf{v}$ is less than $\frac{\pi}{2}$.

$$\vec{u}\cdot\vec{v} = \langle b^2-b, -2b, 2\rangle \cdot \langle b-1, b, b\rangle > 0$$

$$\Rightarrow b^3 - 4b^2 + 3b > 0$$

$$\Rightarrow b(b-1)(b-3) > 0 \qquad b \in (0,1) \cup (3,\infty)$$

$-$ $+$ $-$ $+$

0 1 3

2. Show that if $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ are orthogonal, then the vectors $\mathbf{u}$ and $\mathbf{v}$ must have the same length.

$$(\vec{u}+\vec{v})\cdot(\vec{u}-\vec{v}) = |\vec{u}|^2 + \vec{v}\cdot\vec{u} - \vec{u}\cdot\vec{v} - |\vec{v}|^2 = 0$$

(Distributive law)

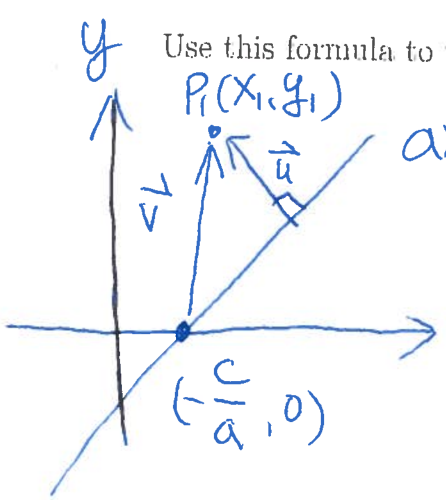
$$\Rightarrow |\vec{u}|^2 - |\vec{v}|^2 = 0$$ (commutative law)

$$\Rightarrow |\vec{u}|^2 = |\vec{v}|^2 \Rightarrow |\vec{u}| = |\vec{v}|$$

3. Use a scalar projection to show that the distance from a point $P_1(x_1, y_1)$ to the line $ax+by+c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

Use this formula to find the distance from the point $(-2, 3)$ to the line $3x - 4y + 5 = 0$.



$ax+by+c=0$, We have $\vec{u} = \langle a, b\rangle$,

$$\vec{v} = (x_1, y_1) - (-\frac{c}{a}, 0) = \langle x_1 + \frac{c}{a}, y_1\rangle$$

Distance from P_1 to the given line

$$= \text{Comp}_{\vec{u}}\vec{v} = \frac{|\vec{u}\cdot\vec{v}|}{|\vec{u}|} = \frac{|a(x_1+\frac{c}{a}) + by_1|}{\sqrt{a^2+b^2}}$$

$$= \frac{|ax_1 + c + by_1|}{\sqrt{a^2+b^2}}$$

So Distance from $(-2,3)$ to line $3x-4y+5=0$

will be : $a=3$, $b=-4$, $x_1=-2$, $y_1=3$

So $d=\dfrac{|3\cdot(-2)-4\cdot(3)+5|}{\sqrt{3^2+(-4)^2}}=\dfrac{13}{5}$ #