Honors Calculus, Math 1451 - HW2

§12.5

2. Given a point $(6,-5,2)$ and a vector $\langle 1,3,-\frac{2}{3}\rangle$.

The vector equation of a line through the given point and parallel to the given vector is

$$\vec{r}(t) = \langle t+6,\ 3t-5,\ -\tfrac{2}{3}t+2 \rangle$$

and parametric equations of this line is

$$x = t+6,\quad y = 3t-5,\quad z = -\tfrac{2}{3}t+2$$

4. Given a point $(0,14,-10)$ and a line $\langle -1+2t, 6-3t, 3+9t \rangle$.

The vector equation of a line through the given point and parallel to the given line is

$$\vec{r}(t) = \langle 2t+0,\ -3t+14,\ 9t-10 \rangle$$

and parametric equations of this line is

$$x = 2t,\quad y = -3t+14,\quad z = 9t-10$$

8. Given two points $(6,1,-3)$ and $(2,4,5)$.

The line. through these two point is parallel to the vector $(6,1,-3)-(2,4,5)=\langle 4,-3,-8\rangle$,

so the parametric equations of this line are

$x=4t+6$; $y=-3t+1$; $z=-8t-3$

(or $x=4t+2$, $y=-3t+4$, $z=-8t+5$) and

the symmetric equations are

$$\frac{x-6}{4}=\frac{y-1}{-3}=\frac{z+3}{-8} \quad \left(\text{or } \frac{x-2}{4}=\frac{y-4}{-3}=\frac{z-5}{-8}\right)$$

10. Given one point $(2,1,0)$ and two vectors $\langle 1,1,0\rangle$ and $\langle 0,1,1\rangle$.

The line which is perpendicular to two given vectors is parallel to the vector $\vec{v}=\langle 1,1,0\rangle\times\langle 0,1,1\rangle$

$=\langle -1,1,-1\rangle$,

so the parametric equations of this line are

$x=-t+2$, $y=t+1$, $z=-t+0$, and

the symmetric equations are

$$\frac{x-2}{-1}=\frac{y-1}{1}=\frac{z}{-1}$$

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16. (a) Given a point (2,4,6) and a plane $x-y+3z=7$.

The line which is perpendicular to the given plane is parallel to the normal vector of the plane: $\langle 1,-1,3\rangle$.

So the parametric equations of this line through the given point are

$$x=t+2,\ y=-t+4,\ z=3t+6.$$

(b) By the parametric equations of the line, the point on the line intersect the given plane can be written as $(t+2, -t+4, 3t+6)$.

- Putting this to the plane xy (i.e. $z=0$)
  $\Rightarrow 3t+6=0 \Rightarrow t=-2 \Rightarrow$ The the point is $(0,6,0)$ which is also an intersecting of the yz plane.

- Putting this to the xz plane (i.e. $y=0$).
  $\Rightarrow -t+4=0 \Rightarrow t=4$. Then the point is $(6, 0, 18)$

20. Given two lines

$L_1: x=1+2t,\ y=3t,\ z=2-t \Rightarrow$ direction: $\langle 2,3,-1\rangle$
point: $(1,0,2)$

$L_2: x=-1+s,\ y=4+s,\ z=1+3s \Rightarrow$ direction: $\langle 1,1,3\rangle$
point $(-1,4,1)$

- First we let the parameters of x and y are the same value, respectively, we have

$$\begin{matrix} 1+2t=-1+s \\ 3t=4+s \end{matrix} \quad \begin{matrix}\Rightarrow \\ \Rightarrow\end{matrix}\begin{cases} 2t-s=-2 \\ 3t-s=4 \end{cases} \quad \text{so, } t=6,\ s=14.$$

Then, we check the parameters of z, we have

$$2-t \underset{t=6}{\underset{\uparrow}{=}} 2-6=-4 \neq 1+3\cdot 14 \underset{s=14}{\underset{\uparrow}{=}} 1+3s$$

$\Rightarrow$ NOT intersecting.

- Since $\langle 2,3,-1\rangle$ is not proportional to $\langle 1,1,3\rangle$

$\Rightarrow$ NOT parallel

Thus, L_1 and L_2 are skew.

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22. Given two lines

$L_1: \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-1}$ and $L_2: \frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3}$

$\Rightarrow$ direction : $\langle 2,2,-1\rangle$ $\Rightarrow$ direction : $\langle 1,-1,3\rangle$

parametric equations:

$x=2t+1$, $y=2t+3$, $z=-t+2$.

- Since $\langle 2,2,-1\rangle$ is NOT proportional to $\langle 1,-1,3\rangle$

$\Rightarrow$ NOT Parallel.

- Using the parametric equations of L_1. and put the parameters of x and y to L_2, we have

$$\frac{(2t+1)-2}{1} = \frac{(2t+3)-6}{-1} \Rightarrow -2t+1 = 2t-3 \Rightarrow t=1.$$

So the point on L_1 is $(3,5,1)$. Put it back to L_2, we have $\frac{3-2}{1} = \frac{5-6}{-1} = \frac{1+2}{3}$. So

L_1 and L_2 are intersecting and the common point is $\underline{(3,5,1)}$.

28. Given a point $(-1,6,-5)$ and a plane $x+y+z+2=0$.

The plane which is parallel to the given plane has the same normal vector of the given plane : $\langle 1,1,1\rangle$

Then the plane through $(-1,6,-5)$ with normal vector $\langle 1,1,1\rangle$ is $x+y+z=0$

36. Given a point $(1,-1,1)$ and a line $x=2y=3z$.

$\Rightarrow$ direction : $\langle 1,\frac{1}{2},\frac{1}{3}\rangle$

point : $(0,0,0)$

So the normal vector of the plane which passes $(1,-1,1)$ and contains the line

is $\vec{n}=\langle 1,\frac{1}{2},\frac{1}{3}\rangle \times \left((1,-1,1)-(0,0,0)\right)$

$=\langle 1,\frac{1}{2},\frac{1}{3}\rangle \times \langle 1,-1,1\rangle = \langle \frac{1}{2}+\frac{1}{3}, -(1-\frac{1}{3}), -1-\frac{1}{2}\rangle$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & \frac{1}{2} & \frac{1}{3} \\ 1 & -1 & 1 \end{vmatrix}$$

$=\langle \frac{5}{6}, -\frac{2}{3}, -\frac{3}{2}\rangle$

or $\langle 5,-4,-9\rangle$

Then the plane is $5x-4y-9z=0$

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48. Given two planes $x+y+z=0$ and $x+2y+3z=1$,

The cosine of the angle between two planes is equal to the cosine of the angle between the normal vectors of two planes.

So $$\cos(\theta)=\frac{\langle 1,1,1\rangle\cdot\langle 1,2,3\rangle}{|\langle 1,1,1\rangle|\,|\langle 1,2,3\rangle|}=\frac{6}{\sqrt{3}\cdot\sqrt{1+4+9}}=\frac{6}{\sqrt{42}}$$

$$\left(\text{or } \frac{\sqrt{42}}{7} \text{ or } \sqrt{\frac{6}{7}}\right).$$

60. Given two points $(2,5,5)$ and $(-6,3,1)$.

The plane consisting all points that are equidistant from two given point has the normal vector

$$(2,5,5)-(-6,3,1)=\langle 8,2,4\rangle \text{ or } \langle 4,1,2\rangle$$

and passes the middle point of two given points:

$$\frac{1}{2}(2,5,5)+\frac{1}{2}(-6,3,1)=(-2,4,3)$$

Then the plane is $4x+y+2z=+2$.

61.

Find the equation of plane with x-intercept a $\Rightarrow$ passes point $P(a,0,0)$

y-intercept b $\Rightarrow$ $Q(0,b,0)$

z-intercept c $\Rightarrow$ $R(0,0,c)$

Now, we have three points $\Rightarrow$ two vectors on plane and one point.

$\overrightarrow{PQ} = (0,b,0) - (a,0,0) = \langle -a, b, 0 \rangle$

$\overrightarrow{PR} = (0,0,c) - (a,0,0) = \langle -a, 0, c \rangle$

So the normal vector of the plane is

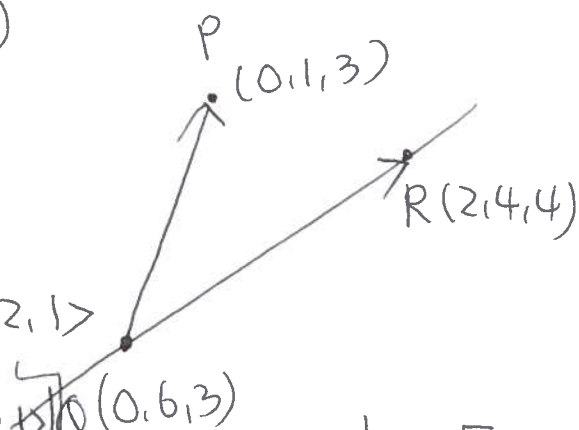
$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = bc\,\vec{i} + ac\,\vec{j} + ab\,\vec{k}.$$

Then the plane is $bcx + acy + abz = abc$.

68. Given a point $P(0,1,3)$ and a line $x = 2t,\ y = 6-2t,\ z = 3+t$.

To find the distance from $P(0,1,3)$ to the line by §12.4 Ex43, we need two points on line:

$t=0$, $Q(0,6,3)$ and $t=1$ $R(2,4,4)$



Then $\vec{b} = \overrightarrow{QP} = (0,1,3) - (0,6,3)$
$= \langle 0, -5, 0 \rangle$

$\vec{a} = \overrightarrow{QR} = (2,4,4) - (0,6,3) = \langle 2, -2, 1 \rangle$

So distance $= \dfrac{|\vec{a} \times \vec{b}|}{|\vec{a}|} = \dfrac{|\langle 0,-5,0\rangle \times \langle 2,-2,1\rangle|}{\sqrt{4+4+1}} = \dfrac{|\langle -5,0,10\rangle|}{3} = \dfrac{5\sqrt{5}}{3}$

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70. Given a point $(-6,3,5)$ and a plane $x-2y-4z=8$.

By formula of the distance between the point and plane,

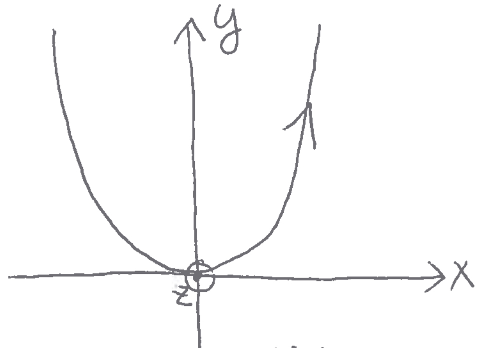
We have

$$d=\frac{|-6-2(3)-4(5)-8|}{\sqrt{1^2+2^2+4^2}}=\frac{40}{\sqrt{21}}.$$

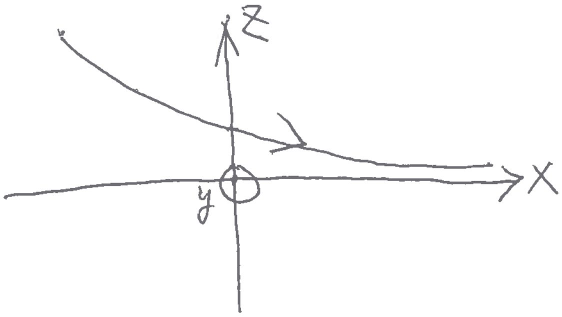
§13.1

20. $x=t,\ y=t^2,\ z=e^{-t}$

When we see this curve from z-direction, we see a parabola.



When we see this curve from y-direction, we see this

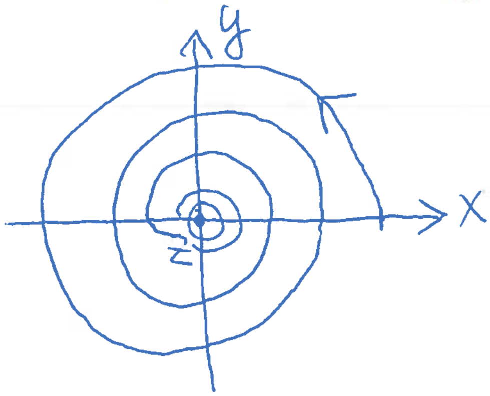


(z is always more than zero)

$\Rightarrow$ II.

22. $x = e^{-t}\cos(10t)$, $y = e^{-t}\sin(10t)$, $z = e^{-t}$.

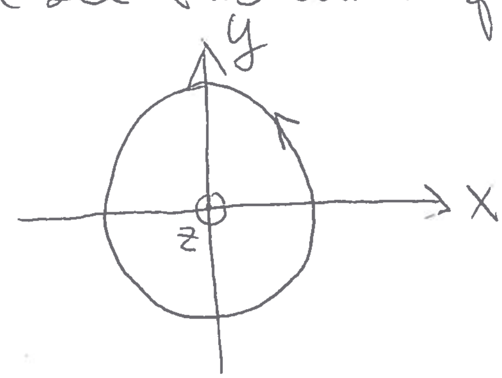
when we see this curve from z-direction, we see a swirl



and z-value is getting smaller and smaller as t is getting bigger. $\Rightarrow$ I.

23. $x = \cos(t)$, $y = \sin(t)$, $z = \sin(5t)$

when we see this curve from z-direction, we see a circle.

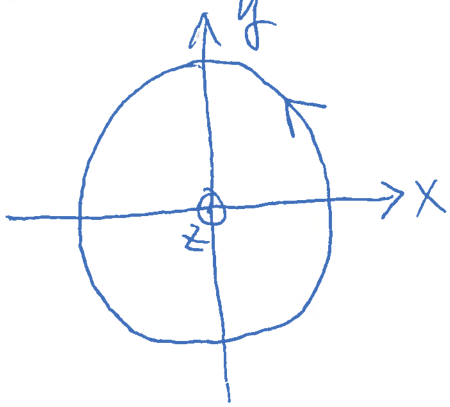


and $z = \sin(5t)$. So It should be V.

which means $|z|$ is bounded by 1.

24. $x=\cos(t)$, $y=\sin(t)$, $z=\ln(t)$.

When we see this curve from z-direction, we get a circle

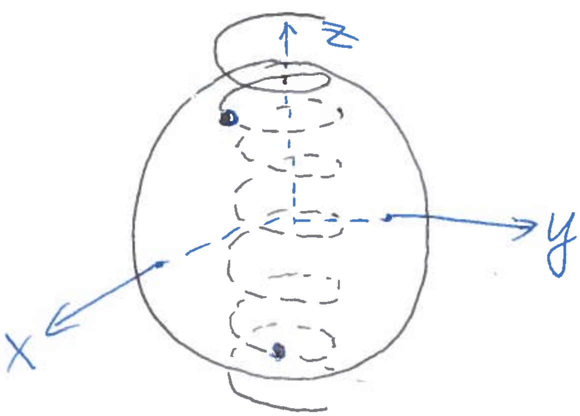


and z is increasing very fast from $t=0$ to $t=1$. then it is increasing NOT ~~that fast~~ after $t=1$. $\Rightarrow$ III

28. To Find the point where the helix $\vec{r}(t)=\langle \sin(t), \cos(t), t\rangle$ intersect the sphere $x^2+y^2+z^2=5$. we put the parameters in the sphere equation. we have

$$\underbrace{(\sin(t))^2+(\cos(t))^2}_{1}+t^2=5 \Rightarrow t^2=4 \Rightarrow t=\pm 2$$

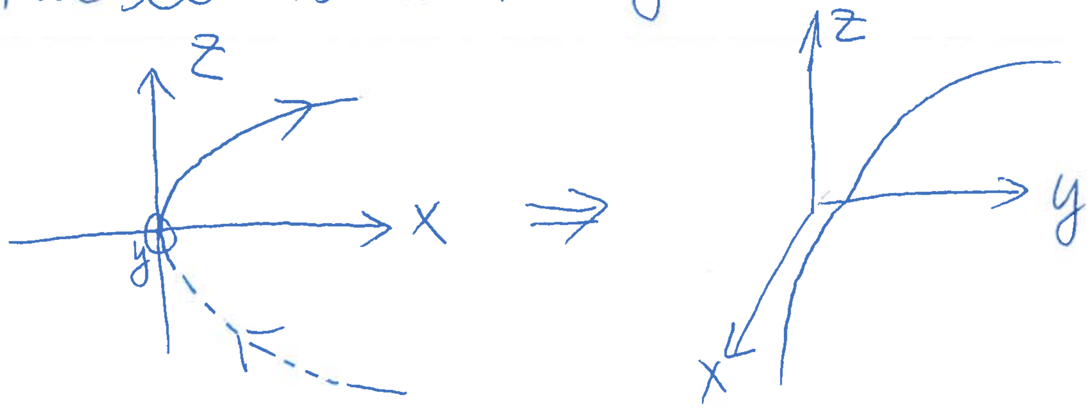
As $t=2$, the point is $(\sin(2), \cos(2), 2)$ and as $t=-2$, the point is $(\sin(-2), \cos(-2), -2)$.



§13.1

30. $\vec{r}(t) = \langle t^2, \ln(t), t \rangle$

· When we see this curve from y-direction, we have.



42. Given two curves $\vec{r}_1(t) = \langle t, t^2, t^3 \rangle$, $\vec{r}_2(t) = \langle 1+2t, 1+6t, 1+14t \rangle$

∘ If particles collide, we have

$$t = 1+2t, \quad t^2 = 1+6t, \quad t^3 = 1+14t$$

$$\Downarrow$$

$$t = -1 \rightarrow t^2 = 1 \neq -5 = 1+6t \Rightarrow \text{NOT collide.}$$

· For intersection, we need two different variables.

$$\vec{r}_1(t) = \langle t, t^2, t^3 \rangle, \quad \vec{r}_2(s) = \langle 1+2s, 1+6s, 1+14s \rangle$$

If two paths intersect, we have

$$t = 1+2s, \quad t^2 = 1+6s, \quad t^3 = 1+14s$$

$$(1+2s)^2 = 1+6s$$

$$\Rightarrow \cancel{1}+4s+4s^2 = \cancel{1}+6s$$

$$4s^2 - 2s = 0$$

$$\Rightarrow s = 0 \text{ or } \tfrac{1}{2} \Rightarrow t = 1 \text{ or } 2$$

check $t=1, s=0$ $\Rightarrow t^3 = 1 = 1+14s$ ✓

check $t=2, s=\frac{1}{2}$ $\Rightarrow t^3 = 8 = 1+14s$ ✓

$\Rightarrow$ The paths have two intersections, (1, 1, 1) (2, 4, 8)