Honors Calculus, Math 1451 - HW1 Solutions

§12.2

20. Let $\vec{a} = 2\vec{i} - 4\vec{j} + 4\vec{k}$, $\vec{b} = 2\vec{j} - \vec{k}$

Then $\vec{a} + \vec{b} = 2\vec{i} - 4\vec{j} + 2\vec{j} + 4\vec{k} - \vec{k} = \underline{2\vec{i} - 2\vec{j} + 3\vec{k}}$

$2\vec{a} + 3\vec{b} = 2(2\vec{i} - 4\vec{j} + 4\vec{k}) + 3(2\vec{j} - \vec{k})$

$= 4\vec{i} - 8\vec{j} + 8\vec{k} + 6\vec{j} - 3\vec{k} = \underline{4\vec{i} - 2\vec{j} + 5\vec{k}}$

$|\vec{a}| = \sqrt{2^2 + (-4)^2 + (4)^2} = \sqrt{36} = \underline{6}$

$|\vec{a} - \vec{b}| = |2\vec{i} - 6\vec{j} + 5\vec{k}| = \sqrt{2^2 + 6^2 + 5^2} = \underline{\sqrt{65}}.$

22. Let $\vec{a} = \langle -4, 2, 4 \rangle$, and $|\vec{a}| = \sqrt{(-4)^2 + 2^2 + 4^2} = 6$

Then the unit vector which has the same direction of $\vec{a}$

is $\dfrac{\vec{a}}{|\vec{a}|} = \langle -\frac{4}{6}, \frac{2}{6}, \frac{4}{6} \rangle = \underline{\langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \rangle}$

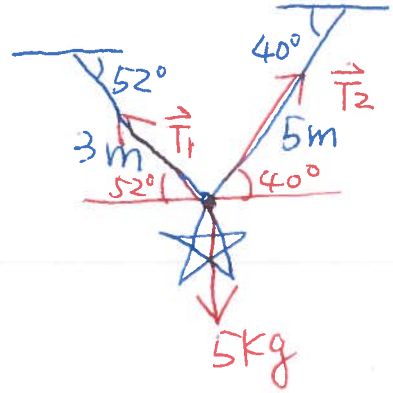
24. Let $\vec{a} = \langle -2, 4, 2 \rangle$ and $|\vec{a}| = \sqrt{(-2)^2 + 4^2 + 2^2} = 2\sqrt{6}.$

Then the vector which has the same direction of $\vec{a}$ and the length is 6 is $6 \cdot \dfrac{\vec{a}}{|\vec{a}|} = 6 \cdot \langle \frac{-2}{2\sqrt{6}}, \frac{4}{2\sqrt{6}}, \frac{2}{2\sqrt{6}} \rangle$

$= \underline{\langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6} \rangle}$

§12.2

32. See the graph. So, we have the system of tensions $\vec{T_1}, \vec{T_2}$:



$$\vec{T_1} = -|\vec{T_1}|\cos(52°)\vec{i} + |\vec{T_1}|\sin(52°)\vec{j}$$

$$\vec{T_2} = |\vec{T_2}|\cos(40°)\vec{i} + |\vec{T_2}|\sin(40°)\vec{j}$$

and $\vec{T_1}+\vec{T_2} = \cancel{5}\ \xrightarrow{} 5\cdot 9.8\ \vec{j}$ —— (1)

Since $\vec{T_1}+\vec{T_2} = \left(-|\vec{T_1}|\cos(52°) + |\vec{T_2}|\cos(40°)\right)\vec{i} + \left(|\vec{T_1}|\sin(52°) + |\vec{T_2}|\sin(40°)\right)\vec{j}$,

By (1), we have

$$\begin{cases} -|\vec{T_1}|\cos(52°) + |\vec{T_2}|\cos(40°) = 0 \Rightarrow |\vec{T_2}| = \dfrac{|\vec{T_1}|\cos(52°)}{\cos(40°)} \\ |\vec{T_1}|\sin(52°) + |\vec{T_2}|\sin(40°) = 5\cdot 9.8 \end{cases}$$

$$\Rightarrow |\vec{T_1}|\sin(52°) + \frac{|\vec{T_1}|\cos(52°)}{\cos(40°)}\cdot\sin(40°) = 5\cdot 9.8$$

$$\Rightarrow |\vec{T_1}| = \frac{5\cdot 9.8}{\sin(52°)+\cos(52°)\cdot\tan(40°)} \cong 3.8325569\ \cancel{(kg)}\cdot 9.8\ (N)$$

and

$$\frac{|\vec{T_2}|\cdot\cos(40°)}{\cos(52°)}\cdot\sin(52°) + |\vec{T_2}|\sin(40°) = 5\cdot 9.8$$

$$\Rightarrow |\vec{T_2}| = \frac{5\cdot 9.8}{\sin(40°)+\cos(40°)\tan(52°)} \cong 3.0801837\ \cancel{(kg)}\cdot 9.8\ (N)$$

$$\Rightarrow \vec{T_1} \approx \left(-2.35955\vec{i} + 3.02009605\vec{j}\right)\cdot 9.8 \quad (N)$$

$$\vec{T_2} \approx \left(2.3595\vec{i} + 1.9799039\vec{j}\right)\cdot 9.8 \quad (N)$$

§12.2

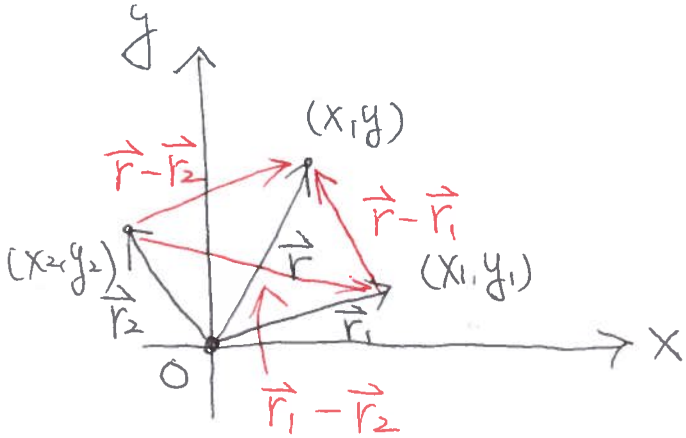
42. Let $\vec{r} = \langle x, y \rangle$, $\vec{r_1} = \langle x_1, y_1 \rangle$, and $\vec{r_2} = \langle x_2, y_2 \rangle$.

To find the set of points (x, y) such that.

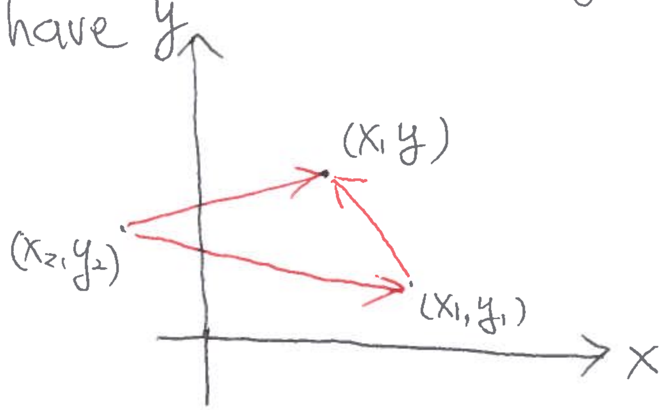
$$|\vec{r} - \vec{r_1}| + |\vec{r} - \vec{r_2}| = K, \text{ where } K > |\vec{r_1} - \vec{r_2}|$$

First, by Parallelogram Law, we assign the initial point of $\vec{r}, \vec{r_1}, \vec{r_2}$ to be origin point $(0, 0)$.

Then we can draw the following graph



Now, we remove the three given vectors $\vec{r}, \vec{r_1}, \vec{r_2}$ on the graph, We have



and the three points (x, y), (x_1, y_1) and (x_2, y_2) form a triangle.

By the triangle inequality, we know that the sum of the length of any two sides of a triangle must be greater than the length of the third side. It means once the three points (x, y), (x_1, y_1), (x_2, y_2) are three vertices of a triangle, we have

$$|\vec{r}-\vec{r}_1|+|\vec{r}-\vec{r}_2| > |\vec{r}_1-\vec{r}_2|.$$

§12.3

1. (a) "$(\vec{a}\cdot\vec{b})\cdot\vec{c}$" is meaningless since $\vec{a}\cdot\vec{b}$ is a scalar and there is no way we can do inner product between a scalar and a vector.

(b) "$(\vec{a}\cdot\vec{b})\vec{c}$" is meaningful since it is a vector ($\vec{c}$) which multiplies by a scalar ($\vec{a}\cdot\vec{b}$)

(c) "$|\vec{a}|(\vec{b}\cdot\vec{c})$" is meaningful since it is a product between two scalars ($|\vec{a}|$ & $(\vec{b}\cdot\vec{c})$)

(d) "$\vec{a}\cdot(\vec{b}+\vec{c})$" is meaningful since it is an inner product between ~~two vectors~~ ($\vec{a}$ & $\vec{b}+\vec{c}$)(the sum of two vectors is a vector.)

(e) $\vec{a}\cdot\vec{b}+\vec{c}$ is meaningless since we can NOT do summation between a scalar ($\vec{a}\cdot\vec{b}$) and a vector ($\vec{c}$).

§12.3

1\. (f) "$|\vec{a}| \cdot (\vec{b}+\vec{c})$" is meaningless since we can NOT do an inner product between a scalar ($|\vec{a}|$) and a vector ($\vec{b}+\vec{c}$).

8\. Let $\vec{a} = 4\vec{j} - 3\vec{k} = \langle 0, 4, -3 \rangle$, and $\vec{b} = 2\vec{i} + 4\vec{j} + 6\vec{k} = \langle 2, 4, 6 \rangle$

Then $\vec{a} \cdot \vec{b} = \langle 0, 4, -3 \rangle \cdot \langle 2, 4, 6 \rangle$

$$= 0 \cdot 2 + 4 \cdot 4 + (-3) \cdot 6 = -4.$$

16\. Given $\vec{a} = \langle \sqrt{3}, 1 \rangle$, $\vec{b} = \langle 0, 5 \rangle$

Since $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$ and $\vec{a} \cdot \vec{b} = 5$.

$|\vec{a}| = \sqrt{3+1} = 2$ and $|\vec{b}| = 5$, Then

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{5}{2 \cdot 5} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}.$$

18\. Given $\vec{a} = \langle 4, 0, 2 \rangle$, $\vec{b} = \langle 2, -1, 0 \rangle$. since $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

and $\vec{a} \cdot \vec{b} = 8 + 0 + 0 = 8$, $|\vec{a}| = \sqrt{16+4} = 2\sqrt{5}$, $|\vec{b}| = \sqrt{4+1} = \sqrt{5}$

Then

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{8}{2\sqrt{5} \cdot \sqrt{5}} = \frac{8}{10} = \frac{4}{5}$$

$$\Rightarrow \theta =$$

24\. (a) Given $\vec{u} = \langle -3, 9, 6 \rangle$ and $\vec{v} = \langle 4, -12, -8 \rangle$

and $\vec{u} \cdot \vec{v} = -12 + (-108) + (-48) < 0$ which means the angle between $\vec{u}$ and $\vec{v}$ is more than $\frac{\pi}{2}$

$\Rightarrow$ ~~Neither.~~ but $\vec{u} = -\frac{4}{3}\vec{v} \Rightarrow \vec{u}$ and $\vec{v}$ are parallel.

DF

24\. Given

(b) $\vec{u} = \vec{i} - \vec{j} + 2\vec{k} = \langle 1, -1, 2 \rangle$ and $\vec{v} = 2\vec{i} - \vec{j} + \vec{k} = \langle 2, -1, 1 \rangle$

Then $\vec{u} \cdot \vec{v} = \langle 1, -1, 2 \rangle \cdot \langle 2, -1, 1 \rangle = 2 + 1 + 2 = 5 > 0$

$\Rightarrow$ The angle between $\vec{u}$ and $\vec{v}$ is more that 0 but less than $\frac{\pi}{2}$

and $\vec{u} \neq d\vec{v}$ for some number "d".

$\Rightarrow$ Neither.

(c) Given $\vec{u} = \langle a, b, c \rangle$ and $\vec{v} = \langle -b, a, 0 \rangle$

Then $\vec{u} \cdot \vec{v} = \langle a, b, c \rangle \cdot \langle -b, a, 0 \rangle = -ab + ab + 0 = 0$

$\Rightarrow$ $\vec{u}$ and $\vec{v}$ are orthogonal.

26\. Given two vectors $\langle -6, b, 2 \rangle$ and $\langle b, b^2, b \rangle$.

These two vectors are orthogonal if and only if

$\langle -6, b, 2 \rangle \cdot \langle b, b^2, b \rangle = 0$

$\Rightarrow -6b + b^3 + 2b = 0 \Rightarrow b^3 - 4b = 0 \Rightarrow b = 0$ or 2 or -2

* $\vec{0}$ is orthogonal to any vector

~~But $b \neq 0$. (if $b = 0$, $\langle b, b^2, b \rangle = \vec{0}$.)~~

~~So $b = 2$ or -2.~~

36\. Given $\vec{a} = \langle 1, 2 \rangle$, $\vec{b} = \langle -4, 1 \rangle$, $\vec{a} \cdot \vec{b} = -2$, $|\vec{a}| = \sqrt{5}$, $|\vec{b}| = 3$

Then the scalar projection of $\vec{b}$ onto $\vec{a}$ is

$$\text{Comp}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{-2}{\sqrt{5}} = -\frac{2}{5}\sqrt{5}$$

and the vector projection of $\vec{b}$ onto $\vec{a}$ is

$$\text{Proj}_{\vec{a}} \vec{b} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \right) \frac{a}{|\vec{a}|} = -\frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} \langle 1, 2 \rangle = \frac{-2}{5} \langle 1, 2 \rangle$$

§12.3

40. Given $\vec{a} = \vec{i} + \vec{j} + \vec{k} = \langle 1,1,1\rangle$ and $\vec{b} = \vec{i} - \vec{j} + \vec{k} = \langle 1,-1,1\rangle$

we have $\vec{a}\cdot\vec{b} = 1-1+1 = 1$ and $|\vec{a}| = \sqrt{3}$.

Then the scalar projection of $\vec{b}$ onto $\vec{a}$ is

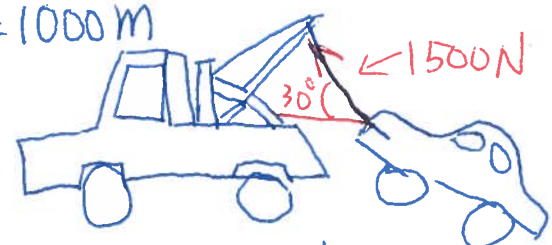
$$\text{Comp}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

and the vector projection of $\vec{b}$ onto $\vec{a}$ is

$$\text{Proj}_{\vec{a}}\vec{b} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\frac{a}{|a|} = \frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}}\langle 1,1,1\rangle = \frac{1}{3}\langle 1,1,1\rangle$$

$$= \langle \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\rangle.$$

46. Given $|\vec{F}| = 1500N$ and $|\vec{D}| = 1\,\text{km} = 1000\,m$



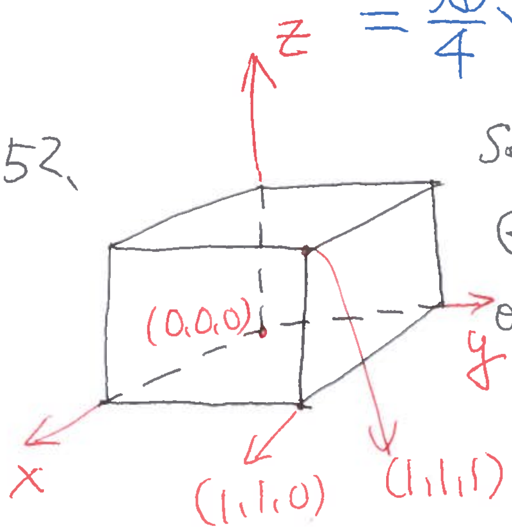
Then $\vec{W} = \vec{F}\cdot\vec{D}$

$$= |\vec{F}||\vec{D}|\cdot\cos\theta = 1500\cdot 1000\cdot\cos(30^\circ)$$

$$= 1.5\times 10^6 \times \frac{\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{4}\times 10^6$$

52.



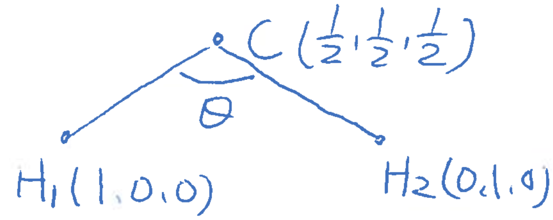
See the graph. Here is a cube with side length 1.
Given a coordinate to represent the vertices of the cube, we have
the vector of a diagonal of this cube is
$(1,1,1) - (0,0,0) = \langle 1,1,1\rangle$
and the vector of a diagonal of one face is
$(1,1,0) - (0,0,0) = \langle 1,1,0\rangle$

Then

the angle between them will be

$$\cos\theta = \frac{\langle 1,1,1\rangle \cdot \langle 1,1,0\rangle}{|\langle 1,1,1\rangle|\ |\langle 1,1,0\rangle|} = \frac{2}{\sqrt{3}\cdot\sqrt{2}} = \frac{2}{\sqrt{6}} = \frac{\sqrt{6}}{3}$$

53. Using hint, we have a tetrahedron with vertices $(1,0,0)$, $(0,1,0)$, $(0,0,1)$, $(1,1,1)$ and centroid $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$. Then we can pick up any two vertices with centroid to find the bond angle which is formed by H—C—H.



See the graph, we have

$\overrightarrow{CH_1} = (1,0,0) - (\frac{1}{2},\frac{1}{2},\frac{1}{2}) = \langle \frac{1}{2}, \frac{-1}{2}, \frac{-1}{2} \rangle$ and

$\overrightarrow{CH_2} = (0,1,0) - (\frac{1}{2},\frac{1}{2},\frac{1}{2}) = \langle -\frac{1}{2}, \frac{1}{2}, \frac{-1}{2} \rangle$

Then $$\cos(\theta) = \frac{\overrightarrow{CH_1}\cdot\overrightarrow{CH_2}}{|\overrightarrow{CH_1}|\,|\overrightarrow{CH_2}|} = \frac{-\frac{1}{4}-\frac{1}{4}+\frac{1}{4}}{\sqrt{\frac{3}{4}}\cdot\sqrt{\frac{3}{4}}} = \frac{-\frac{1}{2}}{\frac{3}{4}} = -\frac{2}{3}$$

$$\Rightarrow \theta = 109.4724\ldots \doteqdot 109.5$$

§12.3

57. By Theorem 3, given two vectors $\vec{a}$ and $\vec{b}$.

Let θ be the angle between $\vec{a}$ and $\vec{b}$, we have

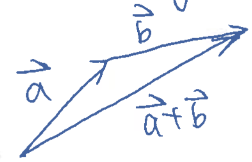
$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

Since $0 \leq |\cos(\theta)| \leq 1$, Then

$|\vec{a}\cdot\vec{b}| = \big||\vec{a}||\vec{b}|\cos(\theta)\big| = |\vec{a}||\vec{b}||\cos(\theta)| \leq |\vec{a}||\vec{b}|$.

58. (a) Given two vectors $\vec{a}$ and $\vec{b}$. Then $\vec{a}$, $\vec{b}$ and $\vec{a}+\vec{b}$ can form a triangle with length of three sides $|\vec{a}|$, $|\vec{b}|$, and $|\vec{a}+\vec{b}|$, respectively.

Thus, by the triangle inequality of a triangle, we have the sum of the length of any two sides must be greater than or equal to the length of the remaining side, that is,



$$|\vec{a}|+|\vec{b}| \geq |\vec{a}+\vec{b}|.$$

(b) By Cauchy-Schwarz Inequality, we have

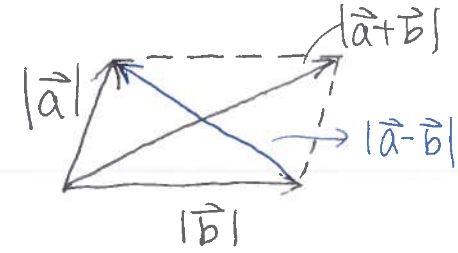
$$|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) \underset{\uparrow\ \text{distributive law}}{=} |\vec{a}|^2 + \vec{b}\cdot\vec{a} + \vec{a}\cdot\vec{b} + |\vec{b}|^2$$

$$\overset{\text{Cauchy-Schwarz}}{\leq} |\vec{a}|^2 + |\vec{b}||\vec{a}| + |\vec{a}||\vec{b}| + |\vec{b}|^2 = (|\vec{a}|+|\vec{b}|)^2$$

$$\Rightarrow |\vec{a}+\vec{b}| \leq |\vec{a}|+|\vec{b}|$$

59.

(a) Given a Parallelogram with two different lengths of sides $|\vec{a}|$, $|\vec{b}|$, so, the lengths of two diagonals are $|\vec{a}+\vec{b}|$ and $|\vec{a}-\vec{b}|$.



So the Parallelogram Law tells us.

the sum of the square of the lengths of two diagonals of a Parallelogram is equal to the sum of the square of the lengths of four sides of this parallelogram.

(b)

$$|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2=(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})+(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$$

$$=|\vec{a}|^2+\vec{b}\cdot\vec{a}+\vec{a}\cdot\vec{b}+|\vec{b}|^2+|\vec{a}|^2-\vec{b}\cdot\vec{a}-\vec{a}\cdot\vec{b}+|\vec{b}|^2$$

$$=2|\vec{a}|^2+2|\vec{b}|^2$$

60. Given two vectors $\vec{u}$, $\vec{v}$. Assume $\vec{u}+\vec{v}$ and $\vec{u}-\vec{v}$ are orthogonal, we have

-2pts

$$(\vec{u}+\vec{v})\cdot(\vec{u}-\vec{v})=0$$

$$\Rightarrow |\vec{u}|^2+\vec{v}\cdot\vec{u}-\vec{u}\cdot\vec{v}-|\vec{v}|^2=0$$

$$\Rightarrow |\vec{u}|^2=|\vec{v}|^2 \Rightarrow |\vec{u}|=|\vec{v}|,$$

that is, $\vec{u}$ and $\vec{v}$ have the same length.

§12.4

4. Given $\vec{a} = \vec{j} + 7\vec{k} = \langle 0, 1, 7 \rangle$ and $\vec{b} = 2\vec{i} - \vec{j} + 4\vec{k} = \langle 2, -1, 4 \rangle$.

Then we have

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 7 \\ 2 & -1 & 4 \end{vmatrix} = 4\vec{i} - (-7\vec{i}) + 14\vec{j} - 2\vec{k}$$

$$= \langle 11, 14, -2 \rangle$$

$$\begin{array}{cccccc} & 2 & -1 & 4 & \\ 2\vec{k} & \vec{i} & \vec{j} & \vec{k} & 4\vec{i} \\ -7\vec{i} & 0 & 1 & 7 & 0 \\ 0 & & & & 14\vec{j} \end{array}$$

and $\vec{a} \cdot (\vec{a} \times \vec{b}) = \langle 0, 1, 7 \rangle \cdot \langle 11, 14, -2 \rangle = 0$.

$\vec{b} \cdot (\vec{a} \times \vec{b}) = \langle 2, -1, 4 \rangle \cdot \langle 11, 14, -2 \rangle = 22 - 14 - 8 = 0$.

13. (a) "$\vec{a} \cdot (\vec{b} \times \vec{c})$" is meaningful and it is a scalar.

2/ for each

(b) "$\vec{a} \times (\vec{b} \cdot \vec{c})$" is meaningless since we can NOT DO cross product between a vector $(\vec{a})$ and a scalar $(\vec{b} \cdot \vec{c})$.

(c) "$\vec{a} \times (\vec{b} \times \vec{c})$" is meaningful and it is a vector.

(d) "$(\vec{a} \cdot \vec{b}) \times \vec{c}$" is meaningless since we can NOT do cross product between a scalar $(\vec{a} \cdot \vec{b})$ and a vector $(\vec{c})$.

(e) "$(\vec{a} \cdot \vec{b}) \times (\vec{c} \cdot \vec{d})$" is meaningless since we can NOT do cross product between two scalars $(\vec{a} \cdot \vec{b})$, $(\vec{c} \cdot \vec{d})$.

(f) "$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$" is meaningful and it is a scalar.

20. let $\vec{a} = \vec{i}+\vec{j}+\vec{k} = <1,1,1>$ and $\vec{b} = 2\vec{i}+\vec{k} = <2,0,1>$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = \vec{i}+\vec{j}-2\vec{k} = <1,1,-2>$$

and $|\vec{a}\times\vec{b}| = \sqrt{1^2+1^2+(-2)^2} = \sqrt{6}$.

Then one of the unit vectors orthogonal to both $\vec{a}, \vec{b}$ is

$<\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}>$. The other one is $<\frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}>$.

30. Given $P(2,1,5)$, $Q(-1,3,4)$, and $R(3,0,6)$

(a) To find a nonzero vector orthogonal to the plane through P, Q, R, it is sufficiently to find the vector which is orthogonal to $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

Then $\overrightarrow{PQ} = (-1,3,4)-(2,1,5) = <-3,2,-1>$ and
$\overrightarrow{PR} = (3,0,6)-(2,1,5) = <1,-1,1>$

We have
$$\overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 2 & -1 \\ 1 & -1 & 1 \end{vmatrix} = \vec{i}+2\vec{j}+\vec{k} = <1,2,1>$$

(or $<-1,-2,-1>$)

(b) Area of $\triangle PQR = \frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}| = \frac{1}{2}\sqrt{1^2+2^2+1^2} = \frac{\sqrt{6}}{2}$.

§12.4

32. Given three points $P(-1,3,1)$, $Q(0,5,2)$, $R(4,3,-1)$.

(a) To find a nonzero vector orthogonal to the plane through P, Q, R, it is sufficiently to find the vector which is orthogonal to $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

Then $\overrightarrow{PQ} = (0,5,2) - (-1,3,1) = \langle 1,2,1 \rangle$ and
$\overrightarrow{PR} = (4,3,-1) - (-1,3,1) = \langle 5,0,-2 \rangle$.

We have

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 5 & 0 & -2 \end{vmatrix} = -4\vec{i} + 7\vec{j} - 10\vec{k} = \langle -4, 7, -10 \rangle$$

(b) Area of $\triangle PQR = \frac{1}{2} |\overrightarrow{PQ} \times \overrightarrow{PR}| = \frac{1}{2} \cdot \sqrt{16+49+100} = \frac{\sqrt{165}}{2}$