Honors Calculus, Exam 1

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Please write your answers clearly and in a logical and well-organized way. Points will be deducted for sloppy work.

Show all working.

Good Luck.

(1) [5] Where does the plane $2x + 4y - z = 5$ intersect the line $x = 4 + 3t$, $y = 3$, $z = -2t$?

(2) [5] Find a *unit* vector orthogonal to the points $(1, -2, 0)$ and $(1, 0, 3)$.

(3) [8] Find the equation of the tangent plane to the surface

$$z = f(x, y) = e^x \cos(x + y)$$

at the point $(0, \frac{\pi}{3})$. Use a tangent plane approximation to estimate $f(\frac{1}{10}, \frac{\pi}{2})$.

(4) [5] Find the distance of the origin to the line given parametrically as $x = 1+2t$,$y = 2 - t$ and $z = t + 1$.

(5) [5] For a perfect gas we have the equation

$$PV = RT$$

where P is pressure, V is volume, T is temperature and R is a constant. If T changes from 400 to 402 degrees and V from 5 to 5.05 m^3 find the approximate change in P via linearization. Note that the answer will involve the constant R.

(6) [7] Show that $u(x, t) = \sin(kx)\sin(\omega t)$ is a solution of the wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\nu^2}\frac{\partial^2 u}{\partial t^2}$$

if $k = \frac{\omega}{\nu}$.

(7) [5] If $x = r\cos\theta$, $y = r\sin\theta$ and $z = f(x, y)$ find:

$$\frac{\partial^2 z}{\partial r^2}$$