Honors Calculus, Exam 1

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Please write your answers clearly and in a logical and well-organized way. Points will be deducted for sloppy work. No calculators are allowed.

Show all working.

Good Luck.

(1) [20] (a) Let $P = (1, 3, 5)$ and $Q = (-1, 1, 1)$. Where does the plane $2x+3y-z = 1$ intersect the line through P in the direction of Q?

(b) Find the equation of the plane passing through the points $P = (2, 1, 0)$, $Q = (3, -1, 1)$ and $R = (4, 1, -1)$.

(c) Find the angle between the vectors $i - 3j + k$ and $-3i + j + 9k$. You may leave your answer as an arcsin, arcos or arctan.

(d) Find the area of the parallelogram spanned by the vectors $\hat{u} = (3, 1, 0)$ and $\hat{v} = (1, -1, 3)$.

(2) [10] (a) If $r(t)$ is a differentiable curve in $\mathbb{R}^3$ show that

$$\frac{d}{dt}(r(t) \times \dot{r}(t)) = r(t) \times \ddot{r}(t)$$

(b) If $r(t)$ is a differentiable curve in $\mathbb{R}^3$ and $\dot{r}(t) \cdot r(t) = 0$ for all t show that $|r(t)|^2$ is constant.

(3) [10] (a) Let $z = \frac{1}{\sqrt{x^2+y^2}}$. Find the linearization of z at the point $(1, 1)$ and use it to estimate the value of z at $(1.01, .98)$.

(b) [5] If $z = \frac{1}{\sqrt{x^2+y^2}}$ show

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

(4) [10] (a) Let $z = f(u) + g(v)$ where $u = x + ct$, $v = x - ct$, c is a constant and x,t are variables.

Using the chain rule show that

$$\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$$

(b) [5] Explain whether or not the following reasoning is correct:

To find the partial derivative of $f(x, y, z)$ with respect to x at the point (a, b, c) we may fix b and c and consider the function $g(x) = f(x, b, c)$. Then $\frac{\partial f}{\partial x}(a, b, c) = \frac{dg}{dx}(a)$ if the the derivative of g exists at $x = a$.