Honors Calculus, Exam 2 Practice.

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(1) [5] Find the direction of maximum increase of the function $f(x, y, z) = x^2 + y^2 - 4z$ at the point $(1, 1, -1)$ and the directional derivative of f in the direction $u = \frac{1}{\sqrt{5}}(2, 1, 0)$.

(2) (a) [7] Find the maximum and minimum values of $f(x, y) = 2x + 3y - x^2 - y^2$ on the closed square in the plane with vertices $(0, 0)$, $(0, 2)$, $(2, 2)$ and $(2, 0)$.

(b) [8] Find the point (or points) on the curve $y^2 = 4 + 3xy$ that are closest to the origin.
Hint: Use Lagrange multipliers and note that $x^2 + y^2 + z^2$ is minimized subject to a constraint if and only if $\sqrt{x^2 + y^2 + z^2}$ is minimized.

(3) [7] Find

$$\iint_R x^2 y^2 dA$$

where R is the region bounded by the lines $y = 2$, $y = 3$, $x = y$ and the y-axis.

(4) (a) [8] Find the integral of the function

$$f(x,y) = \frac{1}{x^2 + y^2 + 2}$$

over the disc $0 \leq x^2 + y^2 \leq R$. (Your answer will involve the constant R).

(b) [2] Letting R tend to infinity find

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{x^2 + y^2 + 2} \, dxdy$$

(5) [7] Find the volume of the region which consists of that part of the sphere $x^2+y^2+z^2 = 2$ which lies above the region $0 \leq x \leq 1$, $0 \leq y \leq \sqrt{1-x^2}$ in the (x,y)-plane, i.e. described by the inequalities

- $0 \leq x \leq 1$,
- $0 \leq y \leq \sqrt{1-x^2}$,
- $0 \leq z \leq \sqrt{2 - x^2 - y^2}$.

(6) [8] Find

$$\iiint_R \frac{1}{\sqrt{x^2 + y^2 + z^2}} dV$$

where R is that part of the solid sphere $x^2 + y^2 + z^2 \leq 2$ that lies above the octant $y \geq 0$, $x \geq 0$ in the (x,y)-plane.