Honors Calculus, Sample Final Exam

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ATTEMPT 6 OUT OF 6 QUESTIONS. ONLY NON-PROGRAMMABLE CALCULATORS ARE ALLOWED. NO GRAPHICAL CALCULATORS ARE TO BE USED.

Please write your answers clearly and in a logical and well-organized way. Points will be deducted for sloppy work.

Good Luck.

(1) Calculate the following line integrals:

(a) $\int_C F ds$ where $F(x,y) = (x^2, xy)$ and C is the part of the parabola between $(0,0)$ and $(1,1)$.

(b) $\int_C F dx$ where $F(x,y) = (x^2, y^2)$ and C is the part of the curve $y = \sin x$ for $0 \leq x \leq \pi$.

(c) $\int_C P dx + Q dy$ where $P(x,y) = y^2$, $Q(x,y) = -x$ and C is the part of the parabola $x = y^2/4$ from $(0,0)$ to $(1,2)$.

(2) Let $F(x, y, z) = (yz + y\cos(xy), xz + x\cos(xy), xy)$.

(a) Show that the integral of F between any two points is independent of the curve between the two points.

(b) Find $\int_C F \cdot dr$ where C is a curve connecting $P = (0, 0, 0)$ to $Q = (\pi, 1, 0)$.

(3) Show that

$$\operatorname{div}(\operatorname{grad}(f) + \operatorname{grad}(g)) = 0$$

if $f,\ g : \mathbb{R}^3 \to \mathbb{R}$ are functions with continuous second order partial derivatives.

(4) A particle starts at the point $(0,0)$ and moves along the x axis to $(1,0)$, then along that part of the circle $x^2 + y^2 = 1$ between $(1,0)$ and $(0,1)$ and then along the y to $(0,0)$ again. Use Green's Theorem to find the work done on the particle by the force field

$$F(x,y) = (x, x^2 + xy)$$

by traversing this curve.

(5) Let S be the sphere

$$x^2 + y^2 + z^2 = 1$$

n the unit normal to the sphere, and F the vector field

$$F(x,y,z) = (\cos^2 x, \sin^2 y, z)$$

Find

$$\int_S F \cdot n dS$$