Honors Calculus, Sample Final 2.

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ATTEMPT ALL QUESTIONS. SHOW ALL WORKING. POINTS WILL NOT BE AWARDED IF WORKING IS NOT SHOWN. NO PROGRAMMABLE CALCULATORS ARE TO BE USED. TIME ALLOWED: 80 MINUTES

Please write your answers clearly and in a logical and well-organized way. Points will be deducted for sloppy work.

GOOD LUCK!

(1) [15 points] In a page or less describe what it means for a series

$$\sum_{n=0}^{\infty} a_n$$

to be: convergent; divergent; absolutely convergent and conditionally convergent. Make sure to mention the role of partial sums and illustrate your discussion with examples.

(2) [10] Explain the truth or falsity of the following two statements, giving reasons and examples:

(i) If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=0}^{\infty} a_n$ converges.

(ii) If $\lim_{n\to\infty} n^{3/2} a_n = 0$ then $\sum_{n=0}^{\infty} a_n$ converges.

(iii) If $\lim_{n\to\infty} \sqrt{n} a_n = 0$ then $\sum_{n=0}^{\infty} a_n$ converges.

(3) [20 points] Determine whether the following series converge. State precisely your reasons.

(a)

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{2^n}$$

(b)

$$\sum_{n=2}^{\infty} \frac{2}{\sqrt{\ln(n)}}$$

(c)

$$\sum_{n=1}^{\infty} \frac{e^n}{(n)!}$$

(d)

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{2n^2+3n+2}}$$

(4) [15 points] (i) Find the Taylor polynomial approximation $T_5(x)$ of degree 5 to $\sin(3x)$ and estimate the accuracy of the approximation for $|x| < .5$.

(ii) Let $T_n(x)$ be the nth order Taylor polynomial for $\sin(x)$ and $R_n(x)$ be the nth order Taylor remainder for the function $\sin(x)$. By estimating the remainder term $R_n(x) = \sin(x) - T_n(x)$ show that $\sin(x)$ equals its Taylor series for all values of x. Briefly discuss why this implies that $\sin(3x)$ equals its Taylor series for all x.

(5) [10 points] (i) Find the power series expansion of the function

$$\frac{2x}{1+x^2}$$

about $a = 0$.

(ii) Using (i) or otherwise find the Taylor series expansion of

$$\ln(1+x^2)$$

about $a = 0$, stating carefully any theorems you may use about integrating or differentiating power series within their radius of convergence.