Honors Calculus, Sample First Midterm (b)

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The test has 6 questions of equal worth. Attempt all questions.

Show all working. Please write your answers clearly and in a logical and well-organized way.

Good luck.

(1) For the function $f(x) = \frac{x^2}{4} - x^2 + 1$ find the set of x values for which:
   (a) f is increasing
   (b) f is decreasing
   (c) find all local maxima and minima.
   (d) find the absolute maximum and minimum.

(2) Show that the equation $2x - 1 - \sin x = 0$ has *exactly* one root.

(3) Find the absolute maximum and minimum values of $\ln(x^2 + x + 1)$ on the closed interval $[-1, 1]$.

(4) Find the following limits (without proof):

(a)
$$\lim_{x\to\infty} x^2 e^{-\sqrt{x}}$$

(b)
$$\lim_{x\to 0} \frac{\cos(x) - 1}{\sin(x)}$$

(c)
$$\lim_{x\to\infty} \frac{3x^2 + 2x + 1}{\sqrt{2x^4 + x^2 + 2}}$$

(d)
$$\lim_{x\to 1} |x - 1| \ln |x - 1|$$

(e)
$$\lim_{x\to 0} \frac{\sin(x^7)}{(2x)^7}$$

(5) Suppose a particle moves on a line so that its position $x(t)$ and velocity $\dot{x}(t)$ satisfy the relation

$$\dot{x}^2(t) + x^2(t) = C$$

where C is a constant. Suppose also that at time $t = 0$, $x(0) = 0$ and $\dot{x}(0) = 3$.

Find the maximum values of $x(t)$, $\dot{x}(t)$ and acceleration $\ddot{x}(t)$.

(6) Suppose a straight line passes through the point $(1, 2)$ in the plane.

(a) Find the value of the slope of such a line that minimizes the distance between the y-intercept and the x-intercept of the line.

(b) Find the value of the slope of such a line that maximizes the area in the first quadrant above the x-axis and under the line.