Honors Calculus, Sample First Midterm (a)

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The test has 7 questions of equal worth. Attempt all questions.

Show all working. Please write your answers clearly and in a logical and well-organized way.

Good luck.

(1) Find the absolute maximum, absolute minimum and all local extremal values of

$$f(x) = \frac{x+1}{x^2+1}$$

on the closed interval $[-1, 1/2]$.

(2) For the function $f(x) = 2x^3 - 9x^2 + 12x + 8$ find the set of x values for which:
(a) f is increasing
(b) f is decreasing
(c) the graph of f is concave up
(d) the graph of f is concave down.
Draw a rough sketch of the graph of f.

(3) A closed cylindrical can is to have a volume of 1600 cubic centimeters. Calculate the dimensions of the can for the surface area to be minimum. Note that both the top and bottom of the can are to be included in the surface area.

(4) The rate of decay of uranium with respect to time is proportional to the mass of uranium.

(a) Write down an equation expressing the statement above.

(b) If it takes 2 years for a piece of uranium decrease in mass 10 grams to 8 grams, what mass will be left after another 3 years?

(5) Suppose f is differentiable on $(-2, 6)$, $f(-1) = 1$ and $-3 \leq f'(x) \leq 3$ for all $x \in (-1, 2)$. Show that

$$-5 \leq f(1) \leq 7$$

State clearly any results that you use.

(6) Suppose that an object of mass m is dropped from rest at a height $2000m$ above the ground and falls to earth (neglect air resistance). Assume that the object satisfies Newton's second law of motion and

$$m\ddot{x} = -mg$$

where g is taken as $10ms^{-2}$.

(a) Find the height of the object above the ground as a function of time. How long does it take to hit the ground?

(b) At the same time as the object in (a) is dropped a projectile is fired upwards from the ground directly beneath the object with a velocity V_0. What is the height of the projectile as a function of time and V_0? Calculate the minimum velocity V_0 needed so that the projectile hits the object in the air.

(7) A particle is moving along the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

(a) Sketch the ellipse, indicating the x and y intercepts i.e. where is crosses the x and y axes.

(b) As the particle passes through the point $(1, \frac{\sqrt{27}}{2})$ its x-coordinate increases at a rate of 1 cm/s.

(i) Is the particle going clockwise or anticlockwise around the ellipse?

(ii) How fast is its y-coordinate changing at that time?

(iii) How fast is its distance from the origin changing at that time?