Honors Calculus, Midterm 2 Sample 1.

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Please write your answers clearly and in a logical and well-organized way. Points will be deducted for sloppy work. Attempt all questions. All questions are of equal value.

(1)[20 points] Evaluate the following definite or indefinite integrals:

(a) $\int_0^1 2x\sqrt{1-x^4}\,dx$

(b) $\int \frac{x^3}{\sqrt{1-x}}\,dx$

(c) $\int \frac{1}{x(\ln x)^2}\,dx$

(d) $\int \frac{1}{2x^2+4}\,dx$

(2)[10 points] Find the area bounded by the curves $y = x^2$, $y = 8x^2$, and $y = 4 - 4x$.

(3) [6 points] Does the improper integral

$$\int_0^1 \frac{1}{x^2} dx$$

converge? If so to what limit?

Does the improper integral

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$$

converge? If so to what limit?

(4) [9 points] The area bounded by $y = \ln x$, $x = 2$, $x = 1$ is rotated about the axis $y = -1$. Find the volume of the resulting solid by both (a) method of cylindrical shells and (b) the method of cross-sectional area.

(5) [5 points] Find (a)

$$\int \frac{1}{(x-1)(x+1)(x+2)} dx$$

(b) [5 points] Show, by comparing a Riemann sum to an integral, that

$$\frac{1}{2} + \frac{1}{3} + \ldots \frac{1}{n+1} < \int_1^{n+1} \frac{1}{x} dx = \ln(n+1)$$