Math 1450, Honor Calculus Practice8, Fall 2016.

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Find the partial fraction decomposition.

1. $f(x) = \dfrac{x^2}{(x-1)(x^2+4x+5)} = \dfrac{\frac{1}{10}}{(x-1)} + \dfrac{Ax+B}{x^2+4x+5}$

Find A $\Rightarrow \lim_{x\to\infty}(x-1)f(x) = \lim_{x\to\infty}\dfrac{x^2}{x^2+4x+5} = \lim_{x\to\infty}\left(\dfrac{1}{10} + \dfrac{Ax(x-1)+B(x-1)}{x^2+4x+5}\right)$

$\Rightarrow \quad 1 = \frac{1}{10} + A \Rightarrow \boxed{A = \frac{9}{10}}$

Find B $\Rightarrow$ let $x=0 \Rightarrow 0 = -\frac{1}{10} + \frac{B}{5} \Rightarrow \boxed{B = \frac{1}{2}}$

2. $f(x) = \dfrac{x}{x^4-1} = \dfrac{x}{(x+1)(x-1)(x^2+1)} = \dfrac{\frac{1}{4}}{x+1} + \dfrac{\frac{1}{4}}{x-1} + \dfrac{Ax+B}{x^2+1}$ ——— (*)

Find A $\Rightarrow \lim_{x\to\infty}(x+1)f(x) = \lim_{x\to\infty}\dfrac{x}{(x-1)(x^2+1)} = \lim_{x\to\infty}\left(\frac{1}{4} + \frac{1}{4}\frac{(x+1)}{(x-1)} + \dfrac{(Ax+B)(x+1)}{x^2+1}\right)$

$\Rightarrow \quad 0 = \frac{1}{4} + \frac{1}{4} + A \Rightarrow \boxed{A = -\frac{1}{2}}$

Find B. $\Rightarrow$ let $x=0$ and put in (*), we have

$0 = \frac{1}{4} - \frac{1}{4} + B \Rightarrow \boxed{B=0}$

3. $f(x) = \dfrac{1}{x(x^2+1)^2} = \dfrac{1}{x} + \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{(x^2+1)^2} =$

$A \to \lim_{x\to\infty} xf(x) \Rightarrow \lim_{x\to\infty}\dfrac{1}{(x^2+1)^2} = \lim_{x\to\infty} 1 + x\left(\dfrac{Ax+B}{x^2+1}\right) + \dfrac{Cx^2+Dx}{(x^2+1)^2} \Rightarrow 0 = 1 + A \Rightarrow A = -1$

$= \dfrac{(x^2+1)^2 + (Ax+B)x(x^2+1) + (Cx+D)x}{x(x^2+1)^2} = \dfrac{x^4+2x^2+1 + Ax^4+Bx^3+Ax^2+Bx+Cx^2+Dx}{x(x^2+1)^2}$

$\Rightarrow \underbrace{(A+1)}_{=0}x^4 + \underbrace{B}_{=0}x^3 + \underbrace{(2+A+C)}_{=0}x^2 + \underbrace{(B+D)}_{=0}x + 1 = 1 \Rightarrow \boxed{A=-1,\ B=0,\ C=-1,\ D=0}$

4. $f(x) = \dfrac{2x^2+1}{x^3-6x^2+11x-6}$

Factorizing $x^3-6x^2+11x-6$, let $g(x)=x^3-6x^2+11x-6$.

Since $g(1)=0$, so $(x-1)$ will be the factor of g.

Using Synthetic Division, we have

$x^3-6x^2+11x-6 = (x-1)(x^2-5x+6)$

$= (x-1)(x-2)(x-3)$.

1	1	−6	+11	−6
		1	−5	+6
	1	−5	6	0

So $f(x) = \dfrac{2x^2+1}{(x-1)(x-2)(x-3)} = \dfrac{A}{x-1} + \dfrac{B}{x-2} + \dfrac{C}{x-3}$.

Find A $\Rightarrow$ $(x-1)f(x) = A + \dfrac{(x-1)}{(x-2)}B + \dfrac{(x-1)}{(x-3)}C$.

$(x-1)f(x) = \dfrac{2x^2+1}{(x-2)(x-3)}$

put $x=1$ $\Rightarrow \dfrac{3}{2} = A+0+0 \Rightarrow A = \dfrac{3}{2}$.

Similarly, $B = \dfrac{9}{1\cdot(-1)} = -9$. $C = \dfrac{19}{2\cdot 1} = \dfrac{19}{2}$.