Math 1450, Honor Calculus Practice3, Fall 2016.

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PSID: ____________ Name: Sol

1. (From 3.7 Exercise 35 in textbook) In the study of ecosystems, predator-prey models are often used to study the interaction between species. Consider populations of tundra wolves, given by $W(t)$, and caribou, given by $C(t)$, in northern Canada. The interaction has been modeled by the equations

$$\frac{dC}{dt} = aC - bCW \qquad \frac{dW}{dt} = -cW + dCW$$

(a) What values of $\frac{dC}{dt}$ and $\frac{dW}{dt}$ correspond to stable populations?

$\frac{dC}{dt} = 0$ & $\frac{dW}{dt} = 0$.

(b) How would the statement "The caribou go extinct" be represented mathematically?

$C = 0$.

(c) Suppose that $a = 500, b = 10, c = 500, d = 1$. Find all population pairs C, W that lead to stable populations. According to this model, is it possible for the two species to live in balance or will one or both species become extinct?

We have $\begin{cases} \frac{dC}{dt} = 500C - 10CW = 0 \Rightarrow 10C(50 - W) = 0 \\ \frac{dW}{dt} = -500W + CW = 0 \end{cases}$ $\Rightarrow C = 0$ or $W = 50$

If $C = 0$, $W = 0$ X

If $W = 50$, $C = \frac{500 \cdot 50}{50} = 500$

This ecosystem will be in balance and $W = 50$, $C = 500$.

2. (From 3.8 Exercise 13 in textbook) A roast turkey is taken from an oven when its temperature has reached 185°F and is placed on a table in a room where the temperature is 75°F.

   (a) If the temperature of the turkey is 150°F after half an hour, what is the temperature after 45 minutes?

   (b) When will the turkey have cooled to 100°F?

Using Newton's Law of Cooling

$$\frac{dT}{dt} = k(T - T_s) \quad \text{where } T_s = 75°F.$$

given $T(0) = 185°F$. $T(30) = 150°F$.

(a) we have, let $y = T - 75$. then

$$y(0) = T(0) - 75 = 185 - 75 = 110$$
$$y(30) = T(30) - 75 = 150 - 75 = 75$$

and $\frac{dy}{dt} = ky \Rightarrow y = y(0)e^{kt} = 110e^{kt}$.

since $y(30) = 75$, $75 = 110e^{30k} \Rightarrow \frac{75}{110} = e^{30k}$

$$\Rightarrow \ln\left(\frac{75}{110}\right) = 30k \Rightarrow k = \frac{1}{30}\ln\left(\frac{75}{110}\right).$$

So $y(45) = 110 \cdot e^{30 \cdot \frac{1}{30}\ln\left(\frac{75}{110}\right)} = 110 \cdot e^{\ln\left(\frac{75}{110}\right)} = 110 \cdot \frac{75}{110}$

$= 75°F$

(b) $100 = y(t) = 110\, e^{\frac{t}{30}\ln\left(\frac{75}{110}\right)}$

take "ln"

$$\Rightarrow \frac{100}{110} = e^{\ln\left(\frac{75}{110}\right)^{\frac{t}{30}}} = \left(\frac{75}{110}\right)^{\frac{t}{30}} \Rightarrow \frac{\ln\left(\frac{100}{110}\right)}{\ln\left(\frac{75}{110}\right)} = \frac{t}{30}$$

$$t = 30 \cdot \frac{\ln\left(\frac{100}{110}\right)}{\ln\left(\frac{75}{110}\right)}$$