Math 1450, Honor Calculus Practice 2, Fall 2016.

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1. For each of the following limits, determine if the limit is computing $f'(a)$ for some function $f(x)$ at the point where $x = a$. If it is, determine $f(x)$, a and $f'(a)$.

i. $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} = \lim_{x\to 0} \frac{(\sqrt{x+4}-2)(\sqrt{x+4}+2)}{x(\sqrt{x+4}+2)} = \lim_{x\to 0} \frac{x}{x(\sqrt{x+4}+2)} = \frac{1}{4}$

We have $f(x)=\sqrt{x+4}$, $a=0$ $\Rightarrow$ so $f'(0)=\frac{1}{4}$

ii. $\lim_{h\to 0} \frac{\frac{2}{3+h}-\frac{1}{3}}{h}$ Assume $f(x)=\frac{2}{3+x}$, and $a=0$, but $f(0)=\frac{2}{3}\neq\frac{1}{3}$

Thus, this fraction isn't written in a way as the definition of derivative of a function f

iii. $\lim_{x\to\pi} \frac{\cos(x)+1}{x-\pi} = \lim_{x\to\pi}\frac{-\cos(x-\pi)+1}{x-\pi} = 0$ $\Rightarrow f'(\pi)=0$

$\boxed{\lim_{x\to 0}\frac{1-\cos(x)}{x}=0}$

We have $f(x)=\cos(x)$, $a=\pi$, $f(a)=-1$

$\boxed{\cos(x-\pi)=\cos(x)\cos(\pi)+\cancel{\sin(\pi)}^{0}\sin(x) \Rightarrow \cos(x-\pi)=-\cos(x)}$

iv. $\lim_{h\to 0} \frac{(1+h)^2-1}{h} = \lim_{h\to 0}\frac{1+2h+h^2-1}{h} = \lim_{h\to 0}\frac{h^2+2h}{h} = \lim_{h\to 0} h+2 = 2$.

We have $f(x)=(1+x)^2$, $a=0$
$f(0)=1$ ✓ $\Rightarrow f'(0)=2$.

v. $\lim_{x\to -1} \frac{\frac{1}{x-1}+\frac{1}{2}}{x+1} = \lim_{x\to -1}\frac{\frac{x+1}{2(x-1)}}{x+1} = \lim_{x\to -1}\frac{1}{2(x-1)} = -\frac{1}{4}$.

We have $f(x)=\frac{1}{x-1}$, $a=-1$
$f(-1)=-\frac{1}{2}$ ✓ $\Rightarrow f'(-1)=-\frac{1}{4}$

vi. $\lim_{h\to 0} \frac{\sin(\pi+h)}{h} = \lim_{h\to 0}\frac{-\sin(h)}{h} = -1$

We have $f(x)=\sin(\pi+x)$,
$a=0$
$\Rightarrow f(0)=0$ ✓

$$\boxed{\begin{aligned}\sin(\pi+h) &= \cancel{\sin(\pi)}^{0}\cos(h)+\cos(\pi)\sin(h)\\ &= -^{1}\sin(h) \quad (\cos(\pi)=-1)\end{aligned}}$$

2. Prove that $\lim_{x \to 0} x^4 \cos(\frac{2}{x}) = 0$.

Since $-1 \le \cos(\frac{2}{x}) \le 1$ for all $x \in \mathbb{R}$. Then.

times x^4 on three of them, we obtain

$$-x^4 \le x^4 \cos(\frac{2}{x}) \le x^4.$$

By squeeze thm, since $\lim_{x \to 0} -x^4 = 0$ and $\lim_{x \to 0} x^4 = 0$

Thus, $\lim_{x \to 0} x^4 \cos(\frac{2}{x}) = 0$

Thinking: How to use the same theorem as question 2 to prove $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$?