Math 1450, Honor Calculus Practice16, Fall 2016.

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1. Find the sum of the following (if possible):

a. $\sum_{k=0}^{\infty}\left(-\frac{3}{4}\right)^k$ $\frac{\text{geometric series}}{|(-\frac{3}{4})|<1}$ $\frac{(-\frac{3}{4})^0}{1-(-\frac{3}{4})} = \frac{1}{\frac{7}{4}} = \frac{4}{7}$

b. $\sum_{k=2}^{\infty}\left(\frac{2}{3}\right)^k$ $\frac{\text{geometric series}}{|\frac{2}{3}|<1}$ $\frac{(\frac{2}{3})^2}{1-(\frac{2}{3})} = \frac{\frac{4}{9}}{\frac{1}{3}} = \frac{4}{3}$

c. $\sum_{k=0}^{\infty}\left(\frac{5}{4}\right)^{k+1}$ $\Rightarrow (\frac{5}{4})>1$, by divergent series test, It diverges

d. $\sum_{k=2}^{\infty}\left(\frac{1}{n}-\frac{1}{n+2}\right) = (\frac{1}{2}-\cancel{\frac{1}{4}})+(\frac{1}{3}-\cancel{\frac{1}{5}})+(\cancel{\frac{1}{4}}-\cancel{\frac{1}{6}})+(\cancel{\frac{1}{5}}-\cancel{\frac{1}{7}})+\cdots = \frac{1}{2}+\frac{1}{3} = \frac{5}{6}$

e. $\sum_{k=0}^{\infty}\frac{6^{k+1}}{7^{k-2}} = \sum_{K=0}^{\infty}\frac{6^3\cdot 6^{k-2}}{7^{k-2}} = 6^3\sum_{K=0}^{\infty}\left(\frac{6}{7}\right)^{k-2} = 6^3\frac{(\frac{6}{7})^{-2}}{1-(\frac{6}{7})} = 6^3\frac{7^2}{6^2}\cdot 7 = 6\cdot 7^3$

2. Determine whether the given series converges or diverges; state which test you are using to determine convergence/divergence and show all work.

a. $\sum_{n=0}^{\infty}\frac{k^2 2^k}{(k+1)!}$ let $a_k = \frac{k^2\cdot 2^k}{(k+1)!}$. $\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\left|\frac{(k+1)^2 2^{k+1}}{(k+2)!}\cdot\frac{(k+1)!}{k^2 2^k}\right| = \lim_{k\to\infty}\left|\frac{(k+1)^2}{k^2}\cdot\frac{1}{k+2}\cdot 2\right| = 0 < 1.$

So, by ratio test, $\sum_{K=0}^{\infty}\frac{k^2\cdot 2^k}{(k+1)!}$ converges

b. $\sum_{n=0}^{\infty}\frac{3^{k+1}}{(k+1)^2 e^k}$ let $a_k = \frac{3^{k+1}}{(k+1)^2 e^k}$. $\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\left|\frac{3^{k+2}}{(k+2)^2 e^{k+1}}\cdot\frac{(k+1)^2 e^k}{3^{k+1}}\right| = \lim_{k\to\infty}\left|\frac{(k+1)^2}{(k+2)^2}\cdot\frac{3}{e}\right| = \frac{3}{e} > 1.$

So, by ratio test, $\sum_{K=0}^{\infty}\frac{3^{k+1}}{(k+1)^2 e^k}$ diverges

c. $\sum_{n=1}^{\infty}\frac{\ln(n)}{n}$

since $\int_1^{\infty}\frac{\ln(x)}{x}dx = \frac{(\ln(x))^2}{2}\Big|_1^{\infty} \to \infty$.

by integral test, $\sum_{n=1}^{\infty}\frac{\ln(n)}{n}$ diverges.

Let $a_n = \frac{2n+1}{\sqrt{n^5+3n^4+1}}$, $b_n = \frac{n}{\sqrt{n^5}} = \frac{1}{n^{\frac{3}{2}}}$ and $\lim_{n\to\infty}\left|\frac{a_n}{b_n}\right| = \lim_{n\to\infty}\left|\frac{2n+1}{\sqrt{n^5+3n^4+1}} \cdot \frac{n^{\frac{3}{2}}}{1}\right| = 2 > 0$

d. $\sum_{n=0}^{\infty} \frac{2n+1}{\sqrt{n^5+3n^4+1}}$ and $\sum b_n = \sum \frac{1}{n^{\frac{3}{2}}}$ converges by p-series; then, by limit Comparison test $\sum \frac{2n+1}{\sqrt{n^5+3n^4+1}}$ converges

e. $\sum_{n=2}^{\infty} \frac{4n^2+1}{n^3-n}$ Let $a_n = \frac{4n^2+1}{n^3-n}$, $b_n = \frac{1}{n}$. $\lim_{n\to\infty}\left|\frac{a_n}{b_n}\right| = \lim_{n\to\infty}\left|\frac{4n^2+1}{n^3-n} \cdot n\right| = 4 > 0$ and $\sum b_n = \sum \frac{1}{n}$ diverges. then, by limit Compar. test, $\sum \frac{4n^2+1}{n^3-n}$ diverges

f. $\sum_{n=2}^{\infty} \frac{4n^2+1}{n^5-n}$ Let $a_n = \frac{4n^2+1}{n^5-n}$, $b_n = \frac{1}{n^3}$. $\lim_{n\to\infty}\left|\frac{a_n}{b_n}\right| = \lim_{n\to\infty}\left|\frac{4n^2+1}{n^5-n} \cdot n^3\right| = 4 > 0$ and $\sum b_n = \sum \frac{1}{n^3}$ converges, then, by limit Compar. test, $\sum \frac{4n^2+1}{n^5-n}$ converges

g. $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$ since $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \neq 0$, so, by divergent test, it diverges.

h. $\sum_{n=0}^{\infty} \frac{n^3}{3^n}$ Let $a_n = \frac{n^3}{3^n}$. $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^3}{3^{n+1}} \cdot \frac{3^n}{n^3}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^3}{n^3} \cdot \frac{1}{3}\right| = \frac{1}{3} < 1$ by, ratio test, $\sum_{n=0}^{\infty} \frac{n^3}{3^n}$ converges.

i. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{4}}}$ diverges by p-series.

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3. Determine if the following series (A) converge absolutely, (B) converge conditionally or (C) diverge.

a. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+3}$

(A) $\sum_{n=1}^{\infty}\left|\frac{(-1)^{n+1}\sqrt{n}}{n+3}\right| = \sum_{n=1}^{\infty}\frac{\sqrt{n}}{n+3}$, using $\sum \frac{1}{\sqrt{n}}$ and limit Compar. test. $\sum_{n=1}^{\infty}\left|\frac{(-1)^{n+1}\sqrt{n}}{n+3}\right|$ diverges.

(B) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sqrt{n}}{n+3}$, let $a_n = \frac{\sqrt{n}}{n+3}$. $\lim_{n\to\infty} a_n = 0$. $a_n > a_{n+1} > 0$

by A.S.T. It converges

$\Rightarrow$ a. is convergent conditionally.

b. $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$

c. $\sum_{n=0}^{\infty} \frac{(-1)^n 4n}{3n^2+2n+1}$

d. $\sum_{n=0}^{\infty} \frac{(-1)^n 3}{\sqrt{3n^2+2n+1}}$

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e. $\sum_{n=0}^{\infty} \frac{(-1)^n 3n}{\sqrt{3n^2+2n+1}}$

3.

b. (A) $\sum_{n=1}^{\infty} \left| \frac{\cos \pi n}{n^2} \right|$. since $\left| \frac{\cos(\pi n)}{n^2} \right| < \frac{1}{n^2}$ and $\sum \frac{1}{n^2}$ converges by p-series,

So, $\sum_{n=1}^{\infty} \left| \frac{\cos(\pi n)}{n^2} \right|$ converges and then

$\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n^2}$ converges absolutely.

c. (A) $\sum_{n=0}^{\infty} \left| \frac{(-1)^n 4n}{3n^2+2n+1} \right| = \sum_{n=0}^{\infty} \frac{4n}{3n^2+2n+1}$ using $\sum \frac{1}{n}$ and limit compar. test.

We have $\lim_{n\to\infty} \left| \frac{4n}{3n^2+2n+1} \div \frac{1}{n} \right| = \frac{4}{3} > 0 \Rightarrow \sum \frac{4n}{3n^2+2n+1}$ diverges.

(B) $\sum_{n=0}^{\infty} \frac{(-1)^n 4n}{3n^2+2n+1}$. let $a_n = \frac{4n}{3n^2+2n+1}$. $\lim_{n\to\infty} a_n = 0$ and $a_n > a_{n+1} > 0$

By A.S.T. $\sum_{n=0}^{\infty} \frac{(-1) 4n}{3n^2+2n+1}$ converges conditionally.

d. (A) $\sum_{n=0}^{\infty} \left| \frac{(-1)^n 3}{\sqrt{3n^2+2n+1}} \right| = \sum_{n=0}^{\infty} \frac{3}{\sqrt{3n^2+2n+1}}$ using $\sum \frac{1}{n}$ and lim compar. test.

We have $\lim_{n\to\infty} \left| \frac{3}{\sqrt{3n^2+2n+1}} \div \frac{1}{n} \right| = \sqrt{3} > 0 \Rightarrow \sum \frac{3}{\sqrt{3n^2+2n+1}}$ diverges.

(B) $\sum_{n=0}^{\infty} \frac{(-1)^n 3}{\sqrt{3n^2+2n+1}}$. let $a_n = \frac{3}{\sqrt{3n^2+2n+1}}$, $\lim_{n\to\infty} a_n = 0$. and $a_n > a_{n+1} > 0$

By A.S.T. $\sum_{n=0}^{\infty} \frac{(-1)^n 3}{\sqrt{3n^2+2n+1}}$ converges conditionally

e (C) $\sum_{n=0}^{\infty} \frac{(-1)^n 3n}{\sqrt{3n^2+2n+1}}$ let $a_n = \frac{3n}{\sqrt{3n^2+2n+1}}$. $\lim_{n\to\infty} a_n = \sqrt{3} \neq 0$.

by divergent test, $\sum \frac{(-1)^n 3n}{\sqrt{3n^2+2n+1}}$ diverges.