Math 1450, Honor Calculus Practice10, Fall 2016.
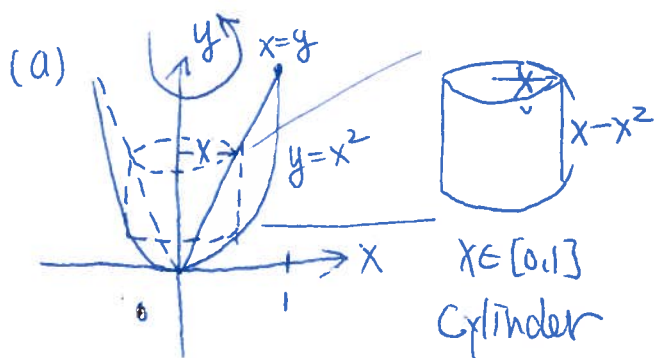
October 14, 2016

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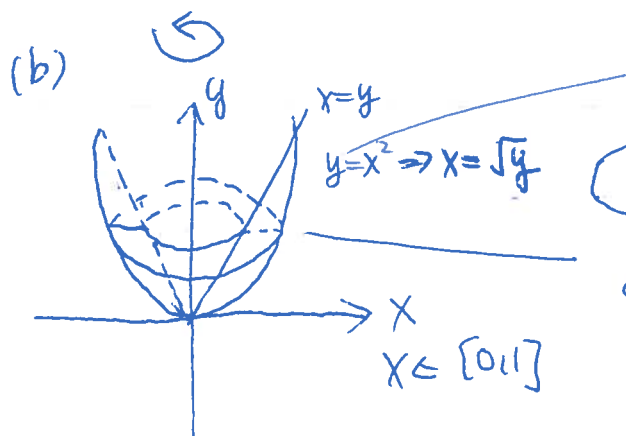
(1) Let V be the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.

   (a) Find V by method of the cylindrical shells.

   (b) Find V by washer method.

(a)

$$V = \int_0^1 2\pi x \cdot (x - x^2)\, dx$$

$$= 2\pi \int_0^1 x^2 - x^3\, dx$$

$$= 2\pi \cdot \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_0^1 = 2\pi \left(\frac{1}{3} - \frac{1}{4} \right) = \frac{\pi}{6}$$
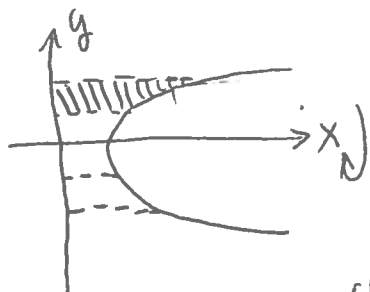
$x \in [0,1]$
Cylinder

(b)

$x = y$
$y = x^2 \Rightarrow x = \sqrt{y}$

$x \in [0,1]$

disk.

$$V = \int_0^1 \pi \left((\sqrt{y})^2 - (y^2)^2 \right) dy$$

$$= \pi \int_0^1 y - y^2\, dy$$

$$= \pi \cdot \left[\frac{y^2}{2} - \frac{y^3}{3} \right] = \frac{\pi}{6}$$

1

(2) Let V be the volume of the solid obtained by rotating about the x-axis the region bounded $x = 1 + y^2$ and $x = 0, y = 1, y = 2$.
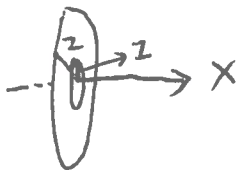
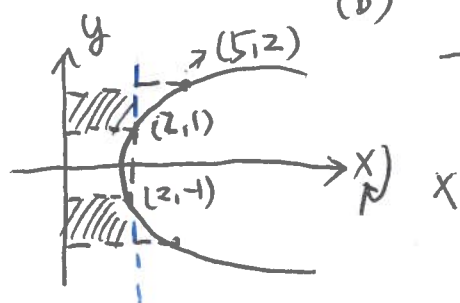(a) Find the integral by method of the cylindrical shells for V.

(b) Find the integral by washer method for V.

(c) Based on (a) and (b), which method will you prefer to find V ?

(a)

$1 + y^2$

$y \in [1, 2]$

$V = \int_1^2 2\pi y (1 + y^2) \, dy$

(b)

$(5, 2)$

$(2, 1)$

$(2, -1)$

$V = \int_0^2 \pi(4-1) \, dx + \int_2^5 \pi(4 - (\sqrt{x-1})^2) \, dx$
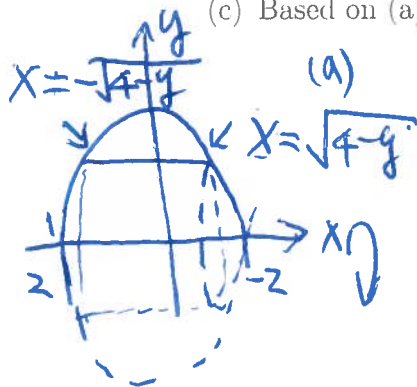
(3) Let V be the volume of the solid obtained by rotating about the x-axis the region bounded $y = 4 - x^2$ and $y = 0$.
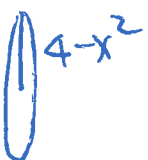
(a) Find the integral by method of the cylindrical shells for V.

(b) Find the integral by washer method for V.

(c) Based on (a) and (b), which method will you prefer to find V ?

$X = -\sqrt{4-y}$

(a)

$X = \sqrt{4-y}$

$\sqrt{4-y} - (-\sqrt{4-y})$

$y \in [0, 2]$

$V = \int_0^4 2\pi y \left(2\sqrt{4-y} \right) \, dy$

(b)

$4 - x^2$

$x \in [-2, 2]$

$V = \int_{-2}^2 \pi(4 - x^2)^2 \, dx$

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