Honors Calculus, Math 1450- Assignment 8, due Friday November 18)

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Sequences and Series

(1) (a) Evaluate

$$\sum_{n\geq 2} \frac{1}{(\sqrt{3})^n}$$

(b) Find the values of x for which

$$\sum_{n=1}^{\infty} |x-1|^n$$

converges and for each x for which the infinite series converges evaluate the sum.

(2) Show that if $|x| < 1$ then

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \ + (-1)^n x^{2n} +$$

and hence evaluate

$$\sum_{n=0}^{\infty} (-1)^n (\frac{1}{4})^{2n+1}$$

(3) Do the following series converge?

(a)

$$\sum_{n>1} \frac{\ln n}{n}$$

(b)

$$\sum_{n>1} \frac{3n(n-1)}{n^2+1}$$

(c)

$$\sum_{j=1}^{\infty} \frac{\cos(\pi j)}{j^2}$$

(d)

$$\sum_{n=0}^{\infty} \frac{\sqrt{n}}{n^2+1}$$

(4) (a) Question 58, Section 11.2

Determine whether the following series are convergent or not. An integral test may be useful.

(b)

$$\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$$

Hint: Consider $\frac{d}{dx} \ln \ln x$

(c)

$$\sum_{n=2}^{\infty} \frac{1}{n\ln(n)^2}$$

Hint: Consider $\frac{d}{dx} (\ln x)^{-1}$

(5) Test the following series for convergence.

(a)

$$\sum_{n=3}^{\infty} \frac{1}{n^2 - 7}$$

(b)

$$\sum_{n=1}^{\infty} \frac{n}{n^4 + 2}$$

(c)

$$\sum_{n=1}^{\infty} \frac{n^2}{e^n}$$

(d)

$$\sum_{n=1}^{\infty} \frac{2^n + 3}{3^n + n}$$

(e)

$$\sum_{n=1}^{\infty} \frac{n}{(n^3 + n)^{1/2}}$$

(f)

$$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$$