Honor Calculus, Math 1450 - Assignment 7 - Solution

Q1.

- $3+4i$

$|3+4i| = \sqrt{3^2+4^2} = 5$

Argument: $\tan\theta = \cancel{\dfrac{3/5}{4/5}} = \cancel{\dfrac{3}{4}}$. $= \dfrac{4/5}{3/5} = \dfrac{4}{3}$ $\quad (3+4i)^{-1} = \dfrac{3-4i}{25}$

- $(3+4i)^{-1} = \dfrac{1}{5e^{i\theta}} = \dfrac{1}{5}e^{-i\theta} \Rightarrow |(3+4i)^{-1}| = \dfrac{1}{5}$ and

argument will ~~be $-\theta$ where $\tan\theta = \frac{3}{4}$~~ $\arctan(-\frac{4}{3})$ ~~$\tan(-\theta) = -\tan\theta = -\frac{3}{4}$~~ $= -\frac{4}{3}$

sin +	All +
tan +	cos +

- $(1-i)^5$.

since $1-i = \sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right) = \sqrt{2}\, e^{\frac{7}{4}\pi i}$

Then $(1-i)^5 = \left(\sqrt{2}\, e^{\frac{7}{4}\pi i}\right)^5 = 4\sqrt{2} \cdot e^{\frac{35}{4}\pi i} = 4\sqrt{2}\, e^{\frac{3\pi}{4} i}$

$|(1-i)^5| = 4\sqrt{2}$, argument: $\frac{3\pi}{4}$.

- $2+3i$

$|2+3i| = \sqrt{2^2+3^2} = \sqrt{13}$.

Argument: θ where $\tan\theta = \dfrac{3}{2}$

Q2.

- $x^2+4=0 \Rightarrow x^2 = -4 \Rightarrow x = \pm 2i$.

Q2. • $x^2+x+1=0$

$$\Rightarrow x = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2}.$$

• $x^4+x^2+1=0$

$$\Rightarrow x^2 = \frac{-1 \pm \sqrt{3}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i = e^{\frac{2}{3}\pi i} \text{ or } e^{\frac{4}{3}\pi i}$$

$$x = \pm e^{\frac{\pi}{3}i}, \pm e^{\frac{2\pi}{3}i}$$

Q3

• $\bar{z} = -z$

let $z = a+ib$, $a, b \in \mathbb{R}$. Then $\bar{z} = a-ib$, $-z = -a-ib$ $\Rightarrow$

$$a-ib = -a-ib \Rightarrow a = -a \Rightarrow a = 0.$$

$\Rightarrow z = ib$. $b \in \mathbb{R}$.

• $|z| < 1$.

let $z = re^{i\theta}$, $r \in \mathbb{R}$. $0 \le \theta < 2\pi$.

Then $|z| = |re^{i\theta}| \underset{|e^{i\theta}|=1}{=} |r| < 1 \Rightarrow |r| < 1$.

• $|z-(1+i)| < 4$

let $z = re^{i\theta}$, then $|z-(1+i)| = |re^{i\theta} - \sqrt{2}e^{\frac{\pi}{4}i}| = |r-\sqrt{2}| < 4$.

$\Rightarrow |r-\sqrt{2}| < 4$.

Q4

let $z = a+ib$, $a, b \in \mathbb{R}$.

Then $\frac{z+\bar{z}}{2} = \frac{a+ib+(a-ib)}{2} = a$ which is real part of z.

and $\frac{z-\bar{z}}{2i} = \frac{a+ib-(a-ib)}{2i} = b$ which is imaginary part of z.

Q4. $z^3 = 1 = e^{i(2k\pi)}$, $k \in \mathbb{N}$.

Then $z = e^{i\left(\frac{2k\pi}{3}\right)}$ $k \in \mathbb{N}$.

$\Rightarrow \quad z = e^{i\frac{2\pi}{3}}, e^{i\frac{4\pi}{3}}, e^{i\frac{6\pi}{3}}, \dots$

$= -\frac{1}{2}+\frac{\sqrt{3}}{2}i, \ -\frac{1}{2}-\frac{\sqrt{3}}{2}i, \ 1.$

Q5. $z^n - 1 = 0 \Rightarrow (z-1)(z^{n-1}+z^{n-2}+\dots+z^2+z+1) = 0$

Since $z-1 \neq 0 \Rightarrow (z^{n-1}+z^{n-2}+\dots+z^2+z+1) = 0$.