Math 1431, Calculus I Test 4 Review, Spring 2015.

Shang-Huan Chiu

April 24, 2015

1 Differentiation and Integration Tables

Assume n, a, and C are constants. $(\dagger) 1 - x^2 > 0$. $(\ddagger) x^2 - 1 > 0$

Function	Derivative of given function
x^n, $n \neq 0$	$n \cdot x^{n-1}$
$\ln(x)$	$\frac{1}{x}$
e^x	e^x
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\sec(x)$	$\sec(x)\tan(x)$
$\cot(x)$	$-\csc^2(x)$
$\csc(x)$	$-\csc(x)\cot(x)$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sinh(x)$
$\arcsin(x)$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan(x)$	$\frac{1}{1+x^2}$
$\text{arcsec}(x)$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
a^x,$a > 0$	$a^x \ln(a)$

Figure 1: Differentiation Table

Function	Integral of given function
x^n	$\int x^n \, dx = \begin{cases} \frac{x^{n+1}}{n+1} + C, & \text{if } n \neq -1; \\ \ln\lvert x\rvert + C, & \text{if } n = -1. \end{cases}$
e^x	$\int e^x \, dx = e^x + C$
$\cos(x)$	$\int \cos(x) \, dx = \sin(x) + C$
$\sin(x)$	$\int \sin(x) \, dx = -\cos(x) + C$
$\sec^2(x)$	$\int \sec^2(x) \, dx = \tan(x) + C$
$\sec(x)\tan(x)$	$\int \sec(x)\tan(x) \, dx = \sec(x) + C$
$\csc^2(x)$	$\int \csc^2(x) \, dx = -\cot(x) + C$
$\csc(x)\cot(x)$	$\int \csc(x)\cot(x) \, dx = -\csc(x) + C$
$\cosh(x)$	$\int \cosh(x) \, dx = \sinh(x) + C$
$\sinh(x)$	$\int \sinh(x) \, dx = \cosh(x) + C$
$\frac{1}{\sqrt{1-x^2}}(\dagger)$	$\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin(x) + C$
$\frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} \, dx = \arctan(x) + C$
$\frac{1}{x\sqrt{x^2-1}}(\ddagger)$	$\int \frac{1}{x\sqrt{x^2-1}} \, dx = \text{arcsec}(x) + C$
a^x,$a > 0$	$\int a^x \, dx = \frac{1}{\ln(a)} a^x + c$

Figure 2: Integration Table

1 Riemann Sum

Given a continuous function f and a partition $P = \{x_0 = a, x_1, x_2, \cdots, x_{n-1}, x_n = b\}$ on $[a, b]$. Then we can estimate $\int_a^b f(x)\,dx$ by Riemann Sum:

$$\sum [(\text{ length of the subinterval}) \times (\text{ value of } f \text{ on this subinterval})]$$

(1) Upper sum (U_f)

$$U_f = \sum [(\text{ length of the subinterval}) \times (\text{ maximum value of } f \text{ on this subinterval})]$$

(2) Lower sum (L_f)

$$L_f = \sum [(\text{ length of the subinterval}) \times (\text{ minimum value of } f \text{ on this subinterval})]$$

(3) $U_f \geq L_f$

(2) Specific points:(left endpoint, right endpoint, midpoint)

$$\text{Sum } = \sum [(\text{ length of the subinterval}) \times (\text{ specific point value of } f \text{ on this subinterval})]$$

2 Basic Integration Properties

Assume f, g are continuous on $[a, b]$ and α, β are constants.

(1) If $a < c < b$, then

$$\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx.$$

(2) The integration value will change of sign if we integrate in the different directions:

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx.$$

(3) The integral from any number to itself is defined to be zero:

$$\int_c^c f(x)\,dx = 0.$$

(4) Linearity of integration:

$$\int_a^b \alpha f(x) + \beta g(x)\,dx = \alpha \int_a^b f(x)\,dx + \beta \int_a^b g(x)\,dx.$$

3 Fundamental Theorem of Calculus

3.1 First Fundamental Theorem of Calculus

Theorem 3.1 *Let f be continuous on $[a, b]$. The function F defined on $[a, b]$ by*

$$F(x) = \int_a^x f(t)\,dt$$

is continuous on $[a, b]$, differentiable on (a, b), and has derivative $F'(x) = f(x)$ for all x in (a, b).

Assume a function f is defined as above in the theorem and function $u(x), v(x)$ are differentiable, we have

(1) If $F(x) = \int_a^x f(t)\,dt$, then

$$F'(x) = f(x).$$

(2) If $F(x) = \int_x^a f(t)\,dt$, so $F(x) = -\int_a^x f(t)\,dt$, then

$$F'(x) = -f(x).$$

(3) If $F(x) = \int_a^{u(x)} f(t)\,dt$, then

$$F'(x) = u'(x) \cdot f(u(x)).$$

(3) If $F(x) = \int_{v(x)}^{u(x)} f(t)\,dt$, there is a constant c such that

$$F(x) = \int_c^{u(x)} f(t)\,dt - \int_c^{v(x)} f(t)\,dt$$

then we have

$$F'(x) = u'(x) \cdot f(u(x)) - v'(x) \cdot f(v(x)).$$

3.2 Second Fundamental Theorem of Calculus

Definition 3.2 Let f be continuous on $[a, b]$. A function is called an antiderivative for f on $[a, b]$ if

$$F \text{ is continuous on } [a, b] \text{ and } F'(x) = f(x) \text{ for all } x \in (a, b).$$

Theorem 3.3 *Let f be continuous on $[a, b]$. If F is any antiderivative for f on $[a, b]$, then*

$$\int_a^b f(t)\,dt = F(b) - F(a).$$

4 Differential

Given $f(a + h)$. we can estimate $f(a + h) - f(a)$ by differential df:

$$f(a + h) - f(a) \approx df = f'(a) \cdot h,$$

then

$$f(a + h) \approx f(a) + f'(a) \cdot h.$$