Math 1431 Exam 4 Review Sol.

1. Find the derivative

a. $y = \ln\sqrt{e^x + 4\sinh(x)} = \ln(e^x + 4\sinh(x))^{\frac{1}{2}} = \frac{1}{2}\ln(e^x + 4\sinh(x))$

$\Rightarrow y' = \frac{1}{2}\,\frac{e^x + 4\cosh(x)}{e^x + 4\sinh(x)}$

b. $y = \sin(\ln(5-x)^6) = \sin(6\ln(5-x))$

$\Rightarrow y' = [\cos(6\ln(5-x))]\cdot[6\ln(5-x)]'$

$= \frac{-6}{5-x}\cos(6\ln(5-x))$

c. $y = x^2 e^{2x} + \ln e^{2x} = x e^{2x} + 2x$ $(f^{-1}(f(a)) = a)$

product

$\Rightarrow y' = e^{2x} + 2x e^{2x} + 2$

d. $y = e^{x^2}\cdot\cosh(3x)$ (product)

$\Rightarrow y' = 2x e^{x^2}\cdot\cosh(3x) + e^{x^2}\cdot 3\cdot\sinh(3x)$

e. $f(x) = \ln(5x^2) + e^{6x} + \arctan(5-2x)$

$f'(x) = \frac{10x}{5x^2} + 6e^{6x} + \frac{(5-2x)'}{1+(5-2x)^2}$

$= \frac{2}{x} + 6e^{6x} + \frac{-2}{1+(5-2x)^2}$

1.f $y=(\tan(x))^{(x^2+7)}$ (Use log. differentiation)

① $\ln y = \ln(\tan(x))^{(x^2+7)} = (x^2+7)\cdot\ln(\tan(x))$

do derivative $\Rightarrow$ $\frac{y'}{y} = 2x\cdot\ln(\tan(x)) + (x^2+7)\,\frac{\sec^2(x)}{\tan(x)}$

"$\times y$" $\Rightarrow$ $y' = \left[2x\cdot\ln(\tan(x)) + (x^2+7)\cdot\frac{\sec^2(x)}{\tan(x)}\right](\tan(x))^{(x^2+7)}$

g. $f(x)=\arctan(2x^3)$ (By formula $[\arctan(u(x))]' = \frac{u'(x)}{1+(u(x))^2}$)

$f'(x) = \frac{6x^2}{1+(2x^3)^2}$

h. $f(x)=\arcsin(3x^2)$ (By formula: $[\arcsin(u(x))]' = \frac{u'(x)}{\sqrt{1-(u(x))^2}}$)

$f'(x) = \frac{6x}{\sqrt{1-(3x^2)^2}}$

i. $y=\cosh(3x)+\sinh(4x)$.

$y' = 3\sinh(3x)+4\cosh(4x)$.

2. Integrate

a. $\int_e^{4e} \frac{1}{x}dx = \ln|x|\Big|_e^{4e} = [\ln(4e)-\ln(e)]$

$= [\ln 4 + \ln e - \ln e] = \ln 4$

2\. b. $\int \left(\frac{\csc^2 x}{2+5\cot x} - e^{9x} \right) dx$ (u-substitution)

let $u = 2+5\cot x$, $du = -5\csc^2 x\,dx \Rightarrow \frac{du}{-5} = \csc^2 x\,dx$

$$= \int \frac{\csc^2 x}{2+5\cot x}\,dx - \int e^{9x}\,dx$$

$$= \int \frac{du}{-5} \cdot \frac{1}{u} - \frac{e^{9x}}{9} + C = -\frac{1}{5}\ln|u| - \frac{e^{9x}}{9} + C$$

$$= -\frac{1}{5}\ln|2+5\cot(x)| - \frac{e^{9x}}{9} + C$$

c. $\int \sec^2(3x)\,dx = \frac{1}{3}\tan(3x) + C$.

d. $\int_0^{\frac{\pi}{4}} \sec(x)\tan(x)\,dx = \sec(x) \Big|_0^{\frac{\pi}{4}} = \sec(\frac{\pi}{4}) - \sec(0)$

$$= \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1$$

e. $\int \frac{x+2}{x^3}\,dx = \int \frac{x}{x^3} + \frac{2}{x^3}\,dx = \int \frac{1}{x^2} + \frac{2}{x^3}\,dx$

$$= -\frac{1}{x} - \frac{1}{x^2} + C$$

f. $\int (3x^3 - 2x^2 + 5)\,dx = \frac{3}{4}x^4 - \frac{2}{3}x^3 + 5x + C$.

g. $\int_1^4 \sqrt{x}\,dx = \int_1^4 x^{\frac{1}{2}}\,dx = \frac{2}{3}x^{\frac{3}{2}} \Big|_1^4 = \frac{2}{3}\,4^{\frac{3}{2}} - \frac{2}{3}\,1^{\frac{3}{2}}$

$$= \frac{2}{3}\cdot 8 - \frac{2}{3} = \frac{14}{3}$$

2. h. $\int_{-8}^{0} \frac{1}{\sqrt{1-x}}\,dx = \int_{1-(-8)}^{1-0} \frac{-1}{\sqrt{u}}\,du = \int_{9}^{1} -u^{-\frac{1}{2}}\,du$

$\boxed{\begin{array}{l} \text{let } u = 1-x \\ du = -dx \end{array}}$

$= +\int_{1}^{9} u^{-\frac{1}{2}}\,du = 2u^{\frac{1}{2}}\Big|_1^9 = 2(9)^{\frac{1}{2}} - 2\cdot 1^{\frac{1}{2}}$

$= 2\cdot 3 - 2\cdot 1 = 4.$

3. If $F'(x) = f(x)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$.

$\hookrightarrow$ F is the anti-derivative of f

4. Given $f(x) = \cos(3x)$, the anti-derivative of f is

$\frac{\sin(3x)}{3} + C$

Graph has y-intercept 3 $\Rightarrow$ as $x=0$, $y=3$. $\Rightarrow$ $C=3$

$\Rightarrow \frac{\sin(3x)}{3} + 3$.

5. Compute: (by Fundamental Theorem of Calculus)

a. $\frac{d}{dx}\int_0^{2-3x} \sin(3t^3)\,dt = (2-3x)' \cdot \sin\left(3(2-3x)^3\right)$

$= -3\cdot\sin\left(3(2-3x)^3\right).$

b. $\frac{d}{dx}\int_{-2x}^{1} \cos(2t^2+1)\,dt = -\frac{d}{dx}\int_1^{-2x} \cos(2t^2+1)\,dt$

$= -(-2x)'\cos(2(-2x)^2+1) = 2\cos(2(-2x)^2+1).$

5. c. $\frac{d}{dx}\int_{4x^2}^{3-5x}\sqrt{t+1}\,dt \underset{\uparrow}{=} \frac{d}{dx}\left(\int_a^{3-5x}\sqrt{t+1}\,dt-\int_a^{4x^2}\sqrt{t+1}\,dt\right)$

let $3-5x>4x^2\geq a$, a is a constant

$$=\frac{d}{dx}\int_a^{3-5x}\sqrt{t+1}\,dt-\frac{d}{dx}\int_a^{4x^2}\sqrt{t+1}\,dt$$

$$=(3-5x)'\sqrt{(3-5x)+1}-(4x^2)'\sqrt{4x^2+1}$$

$$=-5\sqrt{4-5x}-8x\cdot\sqrt{4x^2+1}$$

6. Given $F(x)=\int_3^{x^2}(t+2)\,dt$.

a. $F(\sqrt{3})=\int_3^{(\sqrt{3})^2}(t+2)dt=\int_3^3(t+2)\,dt=0$

b. $F'(x)=(x^2)'(x^2+2)=2x(x^2+2)$ (By F.T.C.)

$\Rightarrow F'(2)=2\cdot2(2^2+2)=4\cdot6=24$

7. Given continuous $f(x)$. Find $f(x)$:

a. $\int_x^2(t+1)f(t)dt=\sin x$

"change range" $\Rightarrow$ $-\int_2^x(t+1)f(t)dt=\sin x$

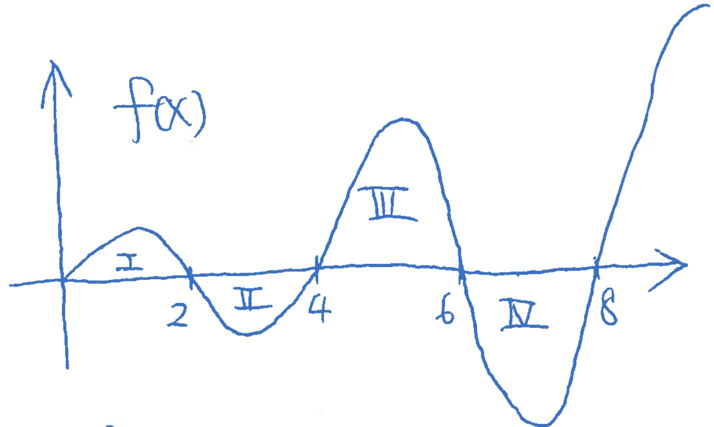
do derivative $\Rightarrow$ on both sides $-(x+1)f(x)=\cos(x)$ $\Rightarrow$ $f(x)=-\frac{\cos(x)}{x+1}$

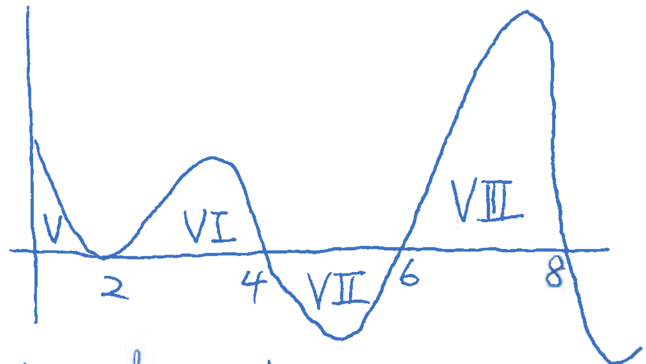
7.

b. $-2x^4-3x^2-6=\int_2^x \frac{f(t)}{t+2}\,dt$

$\xrightarrow[\text{on both sides}]{\text{do } \frac{d}{dx}} -8x^3-6x=\frac{f(x)}{x+2} \Rightarrow \underline{f(x)=(x+2)(-8x^3-6x)}.$

8.





Area of I: 1
" II: 3
III: 5
IV: 7

Area of V: 1.
VI: $\frac{3}{2}$
VII: $\frac{5}{2}$
VIII: 5.

Then

a. $\int_2^8 (f(x)+2g(x))\,dx=\int_2^8 f(x)\,dx+2\int_2^8 g(x)\,dx$

$=\text{area}(\ -\text{II}+\text{III}-\text{IV}\ )+2\cdot\text{area}(\ +\text{VI}-\text{VII}+\text{VIII}\ ).$

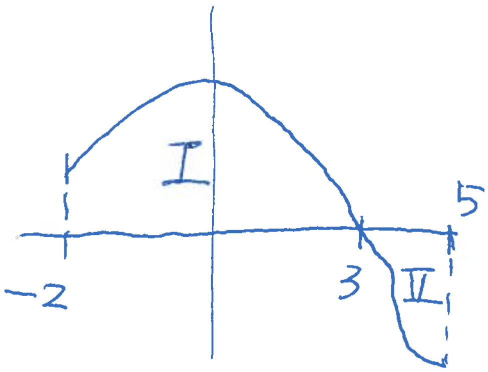
$=\ -3+5-7+2(\ +\frac{3}{2}-\frac{5}{2}+5)=-5+2(4)=\underline{3}$

b. $\int_0^6 (f(x)-g(x))\,dx=\int_0^6 f(x)\,dx-\int_0^6 g(x)\,dx$

$=\text{area}(\text{I}-\text{II}+\text{III})-\text{area}(\text{V}+\text{VI}-\text{VII})$

$=1-3+5-(1+\frac{3}{2}-\frac{5}{2})=3+0=\underline{3}$

9. Given graph $f(x)$ and Area II $= 3$ and $\int_{-2}^{5} f(x)\,dx = 2$.

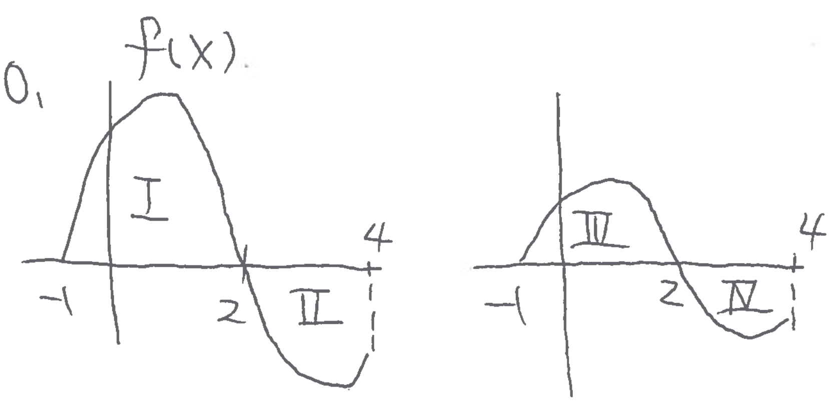


Since $\int_{-2}^{5} f(x)\,dx = \text{area}(\text{I} - \text{II})$

then $2 = \text{area}(\text{I}) - \text{area}(\text{II}) = \text{area}(\text{I}) - 3$.

$\Rightarrow \text{Area}(\text{I}) = 5$.

10.



area I : 2
II : 3
III : 1
IV : 2

Then $\int_{-1}^{4} (-2f(x) + 3g(x))\,dx = -2\int_{-1}^{4} f(x)\,dx + 3\int_{-1}^{4} g(x)\,dx$

$= -2\left(\text{area}(\text{I}-\text{II})\right) + 3\left(\text{area}(\text{III}-\text{IV})\right)$

$= -2\cdot(2-3) + 3(1-2) = -2(-1) + 3(-1) = -1$

11. Given $f(x) = -x^3 + 12$ and partition $P = \{-2, 0, 1, 2\}$.

subinterval	max. value	length	min. value
$[-2, 0]$	$f(-2) = 20$	2	$f(0) = 12$
$[0, 1]$	$f(0) = 12$	1	$f(1) = 11$
$[1, 2]$	$f(1) = 11$	1	$f(2) = 4$.

Upper Sum $= \Sigma[(\text{length}) \times (\text{max. value})] = 2\cdot 20 + 1\cdot 12 + 1\cdot 11 = 63$.

lower Sum $= \Sigma[(\text{length}) \times (\text{min. value})] = 2\cdot 12 + 1\cdot 11 + 1\cdot 4 = 39$

12. Given $f(x)=4-x^2$ and Partition $P=\{-2,-1,0,1,2\}$

Subinterval	length	Value of midpoint
$[-2,-1]$	1	$f(-\frac{3}{2})=\frac{7}{4}$
$[-1,0]$	1	$f(-\frac{1}{2})=\frac{15}{4}$
$[0,1]$	1	$f(\frac{1}{2})=\frac{15}{4}$
$[1,2]$	1	$f(\frac{3}{2})=\frac{7}{4}$

Riemann Sum $=\Sigma[(\text{length})\times(\text{Value})]=1\cdot\frac{7}{4}+1\cdot\frac{15}{4}+1\cdot\frac{15}{4}+1\cdot\frac{7}{4}=11$

13. Given $f(x)=4-x^2$, and partition $P=\{-2,-1,0,1,2\}$

Subinterval	length	value of left hand point.
$[-2,-1]$	1	$f(-2)=0$
$[-1,0]$	1	$f(-1)=3$
$[0,1]$	1	$f(0)=4$
$[1,2]$	1	$f(1)=3$

left hand point

Riemann Sum $=\Sigma[(\text{length})\times(\text{Value})]=1\cdot 0+1\cdot 3+1\cdot 4+1\cdot 3=10$

14. Given $f(x)=\ln(2x-5)+e^{(x-3)}$, and point $(3,1)$.

$f'(x)=\frac{2}{2x-5}+1\cdot e^{x-3}$

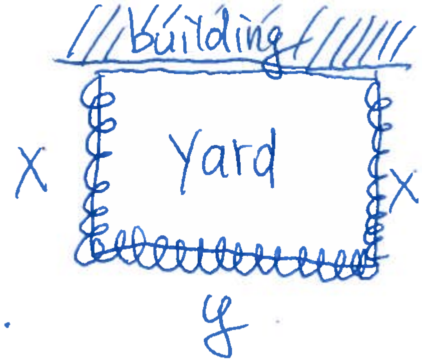
tangent line of f at $(3,1)$: slope: $f'(3)=\frac{2}{6-5}+1\cdot e^0=2+1=3$

$\Rightarrow$ tangent line is $y-1=3(x-3)$

normal line of f at $(3,1)$: slope: $\frac{-1}{f'(3)}=-\frac{1}{3}$

$\Rightarrow$ normal line is $y-1=-\frac{1}{3}(x-3)$

15.
① max. area $\Rightarrow xy$



② The relation : $2x+y=1600$

③ The restriction $\Rightarrow y=1600-2x$.

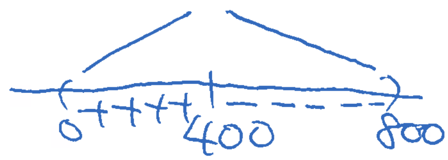
$x>0,\ y>0 \Rightarrow x>0,\ 1600-2x>0$

$\Rightarrow x>0,\ x<800$

④ Let $f(x)=xy=x(1600-2x)=1600x-2x^2$.

$f'(x)=1600-4x \Rightarrow f'(x)=0$ implies $x=400$.

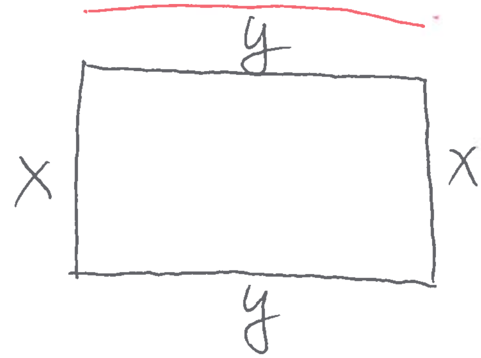
Check number line:



$\Rightarrow$ Max. Value

As $x=400$, $y=800$, Area $=400\times 800=320000\ ft^2$

16. ① min perimeter $=2x+2y$



② The relation: $xy=400$

$\Rightarrow y=\frac{400}{x}$

③ The restriction : $x>0,\ y>0$.

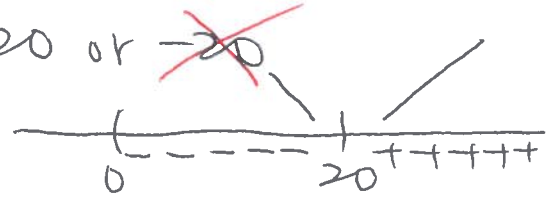
④ Let $f(x)=2x+2y=2x+2\frac{400}{x}$.

$f'(x)=2-\frac{800}{x^2}=\frac{2x^2-800}{x^2} \Rightarrow f'(x)=0$ implies $2x^2-800=0$

($f'(x)$ DNE, $x=0$ (NOT IN DOMAIN))

$\Rightarrow x^2=400 \Rightarrow x=20$ or $\cancel{-20}$

Check number line



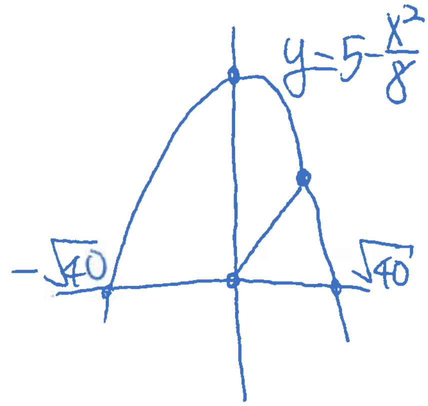
$\Rightarrow$ min. Value

As $x=20$, $y=20 \Rightarrow$ perimeter $=2\cdot 20+2\cdot 20=80$

17. ① Function : point closed to origin.

$\Rightarrow$ the distance is smallest

Let (x, y) be the point on $y = 5 - \frac{x^2}{8}$.



distance: $= d = \sqrt{(x-0)^2 + (y-0)^2}$

$\Rightarrow$ Consider function $d^2 = x^2 + y^2$. (minimize $d \Leftrightarrow$ minimize d^2).

② The relation: $8y = 40 - x^2 \Rightarrow x^2 = 40 - 8y$.

③ The restriction: $y \le 5$, $x \in \mathbb{R}$.

④ Let $f(y) = x^2 + y^2 = 40 - 8y + y^2$, $f'(y) = -8 + 2y$.

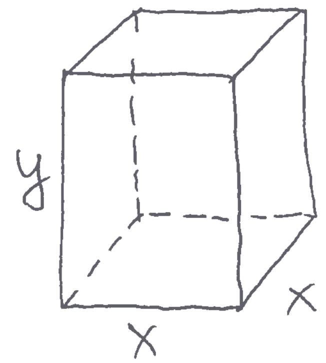
$\Rightarrow f'(y) = 0$ implies $y = 4$. $\quad$ − − − − − − − − | + + + +) $\Rightarrow$ Min. (4, 5)

As $y = 4$, $x^2 = 40 - 32 = 8$, $x = \pm 2\sqrt{2}$

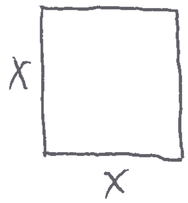
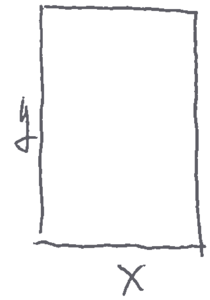
$\Rightarrow$ Two points $(2\sqrt{2}, 4)$ or $(-2\sqrt{2}, 4)$

18. ① Max. Volume, function $= x^2 y$.

② The relation: 600 inch^2 material.



a box with open top.

 $\cdot\, 1 +$  $\cdot\, 4 \Rightarrow x^2 + 4xy = 600$

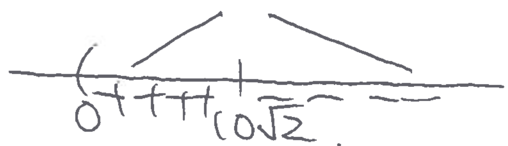
$\Rightarrow y = \dfrac{600 - x^2}{4x}$.

③ The restriction: $x > 0,\ y > 0$.

④ Let $f(x) = x^2 y = x^2 \cdot \dfrac{600 - x^2}{4x} = \dfrac{x}{4}(600 - x^2) = 150x - \dfrac{x^3}{4}$

$f'(x) = 150 - \frac{3}{4}x^2$, $f'(x) = 0$ implies $x^2 = 150 \cdot \frac{4}{3} = 200$

$\Rightarrow x = 10\sqrt{2}$ or $\cancel{-10\sqrt{2}}$

Check number line 

$\Rightarrow$ Max.

$x = 10\sqrt{2}$. $y = \dfrac{600 - (10\sqrt{2})^2}{4 \cdot 10\sqrt{2}} = \dfrac{400}{40\sqrt{2}} = \dfrac{10}{\sqrt{2}} = \dfrac{10}{2}\sqrt{2} = 5\sqrt{2}$

Max. Volume $= x^2 y = (10\sqrt{2})^2 \cdot 5\sqrt{2} = \underline{1000\sqrt{2}}$

19. Use differentials to approximate $\sqrt{63}$. $(f(a+h) \approx f(a) + f'(a) \cdot h)$

① Find $f(x) = \sqrt{x}$. $f'(x) = \frac{1}{2\sqrt{x}}$ ② Pick up a, $a = 64$. (since $\sqrt{64}$ is an integer and is closed to 63)

③ $a+h = 63$, $a = 64 \Rightarrow h = -1$.

④ $\sqrt{63} = f(a+h) \approx f(a) + f'(a) \cdot h = \sqrt{64} + \frac{1}{2\sqrt{64}} \cdot (-1)$

$= 8 + \frac{1}{2 \cdot 8} \cdot (-1) = \frac{127}{16}$.

20. Given $f(x) = x^2 - 3x$, $f'(x) = 2x - 3$.

then $f(1.1) - f(1) \cong f'(1) \cdot h = f'(1) \cdot \frac{1}{10} = (2-3) \cdot \frac{1}{10}$

$= \frac{-1}{10}$.

21. Change degree to radians: $28° \cdot \frac{\pi}{180°} = \frac{28\pi}{180}$.

① Find $f(x) = \tan(x)$. $f'(x) = \sec^2(x)$ ② Pick up $a = \frac{30\pi}{180} = \frac{\pi}{6}$.

③ Since $a+h = \frac{28\pi}{180}$, $a = \frac{\pi}{6} \Rightarrow h = -\frac{2\pi}{180} = -\frac{\pi}{90}$

④ $\tan(28°) = f(a+h) \approx f(a) + f'(a) \cdot h$

$= \tan(\frac{\pi}{6}) + \sec^2(\frac{\pi}{6}) \cdot (-\frac{\pi}{90})$.

$= \frac{1}{\sqrt{3}} + \frac{4}{3} \cdot (-\frac{\pi}{90}) = \frac{1}{\sqrt{3}} - \frac{2\pi}{135}$

22.

a. $\lim_{x\to 0} \frac{1+x-e^x}{x^2} \left(\frac{0}{0}\right) \overset{L'}{=} \lim_{x\to 0} \frac{1-e^x}{2x} \left(\frac{0}{0}\right) \overset{L'}{=} \lim_{x\to 0} \frac{-e^x}{2} = -\frac{1}{2}$

indeterminate form $\left(\frac{0}{0}\right)$.

b. $\lim_{x\to 1} \frac{x+\ln x}{2x^2} \underset{\ln 1 = 0}{\overset{\uparrow}{=}} \frac{1+0}{2} = \frac{1}{2}$ (Need NOT TO APPLY L'Hôpital's Rule)

c. $\lim_{x\to\infty} \left(1+\frac{2}{x}\right)^{2x} (1^\infty) = \lim_{x\to\infty} e^{\ln\left(1+\frac{2}{x}\right)^{2x}} = \lim_{x\to\infty} e^{2x\ln\left(1+\frac{2}{x}\right)}$

exp. function is continuous

$\overset{\downarrow}{=} e^{\lim_{x\to\infty} 2x\ln\left(1+\frac{2}{x}\right)} = e^4$

and

$\lim_{x\to\infty} 2x\ln\left(1+\frac{2}{x}\right) = \lim_{x\to\infty} \frac{2\ln\left(1+\frac{2}{x}\right)}{\frac{1}{x}} \left(\frac{0}{0}\right) \overset{L'}{=} \lim_{x\to\infty} \frac{2\cdot\frac{-2}{x^2}\cdot\frac{1}{1+\frac{2}{x}}}{\frac{-1}{x^2}} = \lim_{x\to\infty} \frac{4}{1+\frac{2}{x}} = 4$

Another Method:

Since $\lim_{f(x)\to\infty} \left(1+\frac{1}{f(x)}\right)^{f(x)} = e$ — $(*)$

Let $f(x) = \frac{x}{2}$, then "$\frac{x}{2}\to\infty$" implies "$x\to\infty$"

Then $\lim_{x\to\infty} \left(1+\frac{2}{x}\right)^{2x} = \lim_{\frac{x}{2}\to\infty} \left(1+\frac{2}{x}\right)^{\frac{x}{2}\cdot 4} = \lim_{\frac{x}{2}\to\infty} \left[\left(1+\frac{2}{x}\right)^{\frac{x}{2}}\right]^4$

$(\cdot)^4$ is continuous and $(*)$

$= \left[\lim_{\frac{x}{2}\to\infty} \left(1+\frac{2}{x}\right)^{\frac{x}{2}}\right]^4 = e^4$.

22. d. $\lim_{x\to 0} \frac{1-\cos x}{x^2} \overset{\left(\frac{0}{0}\right)}{\underset{L'}{=}} \lim_{x\to 0} \frac{\sin(x)}{2x} \overset{\left(\frac{0}{0}\right)}{\underset{L'}{=}} \lim_{x\to 0} \frac{\cos(x)}{2} = \frac{1}{2}$

e. $\lim_{n\to\infty} \frac{\ln(n+4)}{n+2} \overset{\left(\frac{\infty}{\infty}\right)}{\underset{L'}{=}} \lim_{n\to\infty} \frac{\frac{1}{n+4}}{1} = 0$

(n is the variable)

f. $\lim_{n\to\infty} (3n)^{\frac{2}{n}} \overset{\infty^0}{=} \lim_{n\to\infty} e^{\ln (3n)^{\frac{2}{n}}} = \lim_{n\to\infty} e^{\frac{2}{n}\ln(3n)}$

$= e^{\lim_{n\to\infty} \frac{2\ln(3n)}{n}} = e^0 = 1$

exp. function is continuous and $\lim_{n\to\infty} \frac{2\ln(3n)}{n} \overset{\left(\frac{\infty}{\infty}\right)}{\underset{L'}{=}} \lim_{n\to\infty} \frac{2\cdot\frac{3}{3n}}{1} = 0.$

g. $\lim_{n\to\infty} \left(1+\frac{3}{n}\right)^{2n} \overset{1^\infty}{=} \lim_{n\to\infty} e^{\ln\left(1+\frac{3}{n}\right)^{2n}} = \lim_{n\to\infty} e^{2n\ln\left(1+\frac{3}{n}\right)}$

$= e^{\lim_{n\to\infty} 2n\cdot\ln\left(1+\frac{3}{n}\right)} = e^6$

exp. function is continuous and

$\lim_{n\to\infty} 2n\cdot\ln\left(1+\frac{3}{n}\right) \overset{\infty\cdot 0}{=} \lim_{n\to\infty} 2\cdot\frac{\ln\left(1+\frac{3}{n}\right)}{\frac{1}{n}} = \lim_{n\to\infty} \frac{2\cdot\left(\frac{-3}{n^2}\right)\frac{1}{1+\frac{3}{n}}}{\frac{-1}{n^2}} = \lim_{n\to\infty} \frac{6}{1+\frac{3}{n}} = 6.$

22. h. $\lim_{x\to\infty} \frac{x^2}{\ln x} \overset{\frac{\infty}{\infty}}{\underset{L'}{=}} \lim_{x\to\infty} \frac{2x}{\frac{1}{x}} = \lim_{x\to\infty} 2x^2 = \infty \text{ (DNE)}$

i. $\lim_{x\to\infty} (e^{3x}+1)^{\frac{1}{2x}} \overset{\infty^0}{=} \lim_{x\to\infty} e^{\ln (e^{3x}+1)^{\frac{1}{2x}}} = \lim_{x\to\infty} e^{\frac{\ln(e^{3x}+1)}{2x}}$

$= e^{\lim_{x\to\infty} \frac{\ln(e^{3x}+1)}{2x}} = e^{\frac{3}{2}}$

exp function is continuous &

$\lim_{x\to\infty} \frac{\ln(e^{3x}+1)}{2x} \overset{(\frac{\infty}{\infty})}{\underset{L'}{=}} \lim_{x\to\infty} \frac{\frac{3e^{3x}}{e^{3x}+1}}{2} = \lim_{x\to\infty} \frac{3e^{3x}}{2e^{3x}+2} \overset{(\frac{\infty}{\infty})}{\underset{L'}{=}} \lim_{x\to\infty} \frac{9\cancel{e^{3x}}}{6\cancel{e^{3x}}} = \frac{3}{2}$

j. $\lim_{x\to 0} \frac{\arctan(4x)}{x} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 0} \frac{\frac{4}{1+16x^2}}{1} = 4.$