Math 1431

Exam 2 Review

1. Find the following limits (if they exist):

   a. $\lim\limits_{x->0} \frac{\sin 4x}{5x} =$

   b. $\lim\limits_{x->0} \frac{\sqrt{4+x}-2}{x} =$

   c. $\lim\limits_{x->0} \frac{\left(\frac{1}{x+1}-1\right)}{x} =$

   d. $\lim\limits_{x->-3} \frac{x^2+x-6}{x^2-9} =$

   e. $\lim\limits_{x->0} \left(x\left(2-\frac{1}{x}\right)\right) =$

   f. $\lim\limits_{x->0} \frac{2\sin x \cos x}{2x} =$

   g. $\lim\limits_{x->0} \frac{5x}{\tan(9x)} =$

   h. $\lim\limits_{x->0} \frac{\sin(x^2)}{6x} =$

2. Determine if the following are continuous. If the function is not continuous, state the type of discontinuity.

   a. $f(x) = \begin{cases} x^2+1 & x<1 \\ 8 & x=1 \\ x^3 & x>1 \end{cases}$

   b. $f(x) = \begin{cases} 2x^2 & x<2 \\ 8 & x=2 \\ x^3 & x>2 \end{cases}$

   c. $f(x) = \begin{cases} 5-x & x<-2 \\ 7 & x=-2 \\ x^2-5 & x>-2 \end{cases}$

   d. $f(x) = \begin{cases} x^2 & \text{if } x<-1 \\ x+2 & \text{if } x \geq -1 \end{cases}$

3. Let $f(x) = \begin{cases} x+1 & \text{if } x \neq 3 \\ k & \text{if } x=3 \end{cases}$

   For what value of *k* would *f(x)* be continuous at *x=3*?

4. Find A and B so that $f(x)$ is continuous:

$$f(x)=\begin{cases} 6x^2-1 & x<-1 \\ A & x=-1 \\ Bx+3 & x>-1 \end{cases}$$

5. Given $f(x)=\begin{cases} 2x & x>3 \\ x^2-3 & x\le 3 \end{cases}$, find $f'(3)$ if it exists.

6. Find the derivative of the following:

   a. $f(x)=3(2x-1)^4$
   b. $y=\sec^3(2x)$
   c. $f(x)=3\sqrt{x}+\dfrac{5}{x}$
   d. $f(x)=\dfrac{1}{\sqrt{x}}$
   e. $f(x)=\dfrac{\sqrt{x}+2x}{x^2}$
   f. $f(x)=(x^2+2x)^4(x-1)^3$
   g. $f(x)=\dfrac{(x-1)(x+1)}{x+2}$
   h. $y=x\sqrt{(x^3+5x)}$
   i. $f(x)=\dfrac{1+\cos x}{1-\cos x}$
   j. $f(x)=\sin^4(4x^2-6x+1)$
   k. $y=\dfrac{\cot x}{x^2}$
   l. $f(\theta)=\sec\theta-\tan\theta$

7. Find $\dfrac{dy}{dx}$ using implicit differentiation.

   a. $x^2+y^2-4x+3y=7$
   b. $\sin x-\cos y-2=0$
   c. $x^3-xy+y^3=1$
   d. $y\sqrt{x}-x\sqrt{y}=16$
   e. $xy=10$
   f. $x\sin 2y=1$
   g. $x^{2/3}+y^{2/3}=5$

h. $\cos(x+y)=4xy$

8. Use the definition of derivative to find the derivative of the following:

a. $f(x)=3x^2-x+2$

b. $f(x)=\dfrac{2}{x+5}$

c. $f(x)=\sqrt{x+1}$

9. Find $\dfrac{d^3}{dx^3}\left(\dfrac{3}{4}x^4-2x^3+x-10\right)$

10. Find $\dfrac{dy}{dx}$ at $x=-2$ for $y=(4x+1)(1-x)^3$

11. Find the second derivative at the point (-2,1) for $x^2-y^2=3$

12. Use interval notation to give the solution set to:

a. $x(2x-1)(3x+4)\le 0$

b. $x^2-7x+6>0$

13. Find: $\dfrac{d}{dx}\left((2x-5)\left(\dfrac{d}{dx}\left(2x^2+x\right)\right)\right)$

14. A particle is moving along the parabola $y^2 = 4(x+2)$. As it passes through the point (7, 6), its y-coordinate is increasing at the rate of 3 units per second. How fast is the x-coordinate changing at this instant?

15. A man is standing on the top of a 15 foot ladder, which is leaning against a wall. Some scientific minded joker comes up and starts to pull the bottom of the ladder away at a steady rate of 6 ft/min. At what rate is the man on the ladder descending if he remains standing on the top rung when the bottom of the ladder is 9 ft. from the wall?

16. A point moves along the curve $y=2x^2+1$ in such a way that the y value is decreasing at the rate of 2 units per second. At what rate is x changing when x $=\dfrac{3}{2}$?

17. On a morning of a day when the sun will pass directly overhead, the shadow of an 40-ft building on level ground is 30 feet long. At the moment in question, the angle the sun makes with the ground is increasing at the rate of $\pi/1500$ radians/minute. At what rate is the shadow length decreasing?

18. Write the equation of the tangent and normal line to

a. $y^2-x+6=0$ at the point $(15,3)$.

b. $2x^2-6xy+y^2=9$ at the point $(1,-1)$.

19. Use the intermediate value theorem to show that the function $f(x)=2x^5+3x+1$ has a root on the interval $[-1,2]$.

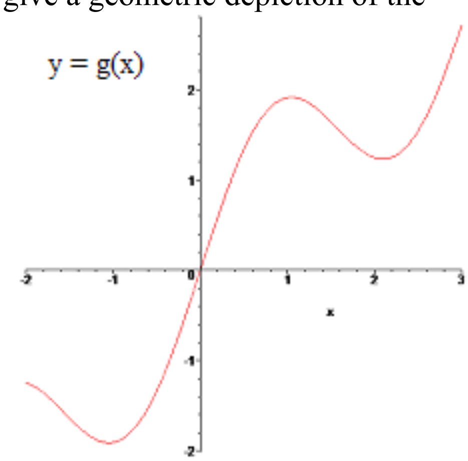
20. Suppose we are given the data in the table about the functions *f* and *g* and their derivatives. Find the following values.

x	1	2	3	4
f(x)	3	2	1	4
f'(x)	1	4	2	3
g(x)	2	1	4	3
g'(x)	4	2	3	1

a) $h(4)\ \text{if}\ h(x) = f(g(x))$ *b)* $h'(4)\ \text{if}\ h(x) = f(g(x))$ *c)* $h(4)\ \text{if}\ h(x) = g(f(x))$

d) $h'(4)\ \text{if}\ h(x) = g(f(x))$ *e)* $h'(4)\ \text{if}\ h(x) = \frac{g(x)}{f(x)}$ *f)* $h'(4)\ \text{if}\ h(x) = f(x)g(x)$

21. Use the plot of the function on the interval [-2,3] to give a geometric depiction of the mean value theorem.



22. Let $f(x) = x^3 - 3x$ be defined on [-1, 1]. Find *c* on (-1, 1) that satisfies the conclusion of the Mean Value Theorem.

23. Consider the function $f(x) = 3x^4 - 20x^3 + 42x^2 - 36x$

a. Show that x = 1 is a critical number of f. Are there any other critical numbers? If so, what are the value(s)?

b. Give the interval(s) of increase and decrease of *f*.