2015, 1, 28

PRINTABLE VERSION

Practice Test 1 Sol

Question 1

Compute $(f \circ g)(x)$, given that $f(x) = \dfrac{4x-3}{2x-1}$ and $g(x) = \dfrac{1}{2x}$.

$$f(g(x)) = \frac{4\left(\frac{1}{2x}\right)-3}{2\left(\frac{1}{2x}\right)-1} = \frac{\frac{2}{x}-3}{\frac{1}{x}-1} \overset{x \neq 0}{=} \frac{2-3x}{1-x} = \frac{3x-2}{x-1}$$

a) $\dfrac{8x^2-4x-1}{2(2x-1)x}$

b) $\dfrac{2x-1}{8x-6}$

c) $\dfrac{3x+1}{x}$

d) $\dfrac{3x-2}{x-1}$

e) $\dfrac{2(4x-3)x}{2x-1}$

Question 2

Find the coordinates of the x-intercept(s) for $f(x) = \dfrac{x^2-x-20}{x^2-8x+15}$.

$(x,0) \Rightarrow$ find x such that $f(x)=0$

$\Rightarrow x^2-x-20=0$

$\Rightarrow (x-5)(x+4)=0$

$\Rightarrow x=5$ or -4.

$(5,0)$ or $(-4,0)$

a) (0,5) and (0,4)

b) (− 3,0) and (− 5,0)

c) (− 4,0)

d) (4,0)

e) (5,0) and (− 4,0)

Question 3

$x \to \infty \Rightarrow f(x) = \frac{3}{2}$

The graph of the function $f(x) = \dfrac{3x^2+12x+12}{2x^2-3x+1}$ has a horizontal asymptote. If the graph crosses this asymptote, give the x-coordinate of the intersection. Otherwise, state that the graph does not cross the

$$\frac{3x^2+12x+12}{2x^2-3x+1} = \frac{3}{2} \Rightarrow \cancel{6x^2}+24x+24 = \cancel{6x^2}-9x+3$$

$$\Rightarrow 33x = -21 \Rightarrow x = -\frac{7}{11}$$

asymptote.

a) $x = -\frac{6}{11}$

b) $x = -\frac{7}{11}$

c) $x = -\frac{10}{11}$

d) $x = -\frac{5}{11}$

e) The graph does not cross the asymptote.

Question 4

Find $f(8)$, $f(-2)$ and $f(-5)$ given

$$f(x) = \begin{cases} 3x^2 + 6 & x \le -3 \\ 4 & -3 < x < 4 \\ -2x - 2 & x \ge 4 \end{cases}$$

$f(8)$, $\boxed{8 \geq 4} \Rightarrow f(8) = -2 \cdot 8 - 2 = -18$

a) $f(8) = 4,\ f(-2) = 18$ and $f(-5) = 81$

$f(-2)$, $\boxed{-3 < -2 < 4} \Rightarrow f(-2) = 4$

b) $f(8) = -18,\ f(-2) = 4$ and $f(-5) = 81$

$f(-5)$, $\boxed{-5 \leq -3} \Rightarrow f(-5) = 3(-5)^2 + 6 = 81$

c) $f(8) = -18,\ f(-2) = 18$ and $f(-5) = 4$

d) $f(8) = 4,\ f(-2) = 4$ and $f(-5) = 81$

e) $f(8) = 198,\ f(-2) = -2$ and $f(-5) = 4$

Question 5

Find the coordinates of the vertex for the following parabola.

$$y = -\frac{1}{4}x^2 + 4x + 6$$

$$y = -\frac{1}{4}(x^2 - 16x + 64) + 6 + \frac{64}{4}$$

a) $(8, 0)$

$$= -\frac{1}{4}(x-8)^2 + 22$$

b) $(0, 6)$

vertex: $(8, 22)$

c) $(8, 6)$

d) $(4, 18)$

e) $(8, 22)$

Question 6

Find the linear function f with $f^{-1}(-6) = 3$ and $f^{-1}(-2) = 4$.

$\Downarrow$ $\Downarrow$

$f(3) = -6$ $f(4) = -2$

a) $f(x) = -\frac{1}{4}x + 3$ $\rightarrow$ X

b) $f(x) = 4x + 18$ $\rightarrow$ X

c) $f(x) = \frac{1}{4}x - 3$ $\rightarrow$ X

d) $f(x) = \frac{1}{4}x + 18$ $\rightarrow$ X

e) $f(x) = 4x - 18$ $\rightarrow$ ✓ ✓

Question 7

Put the equation in standard form for a hyperbola.

$$16x^2 - 9y^2 + 64x + 36y = 116$$

$\Downarrow$

$16x^2 + 64x + \_\_ - 9y^2 + 36y + \_\_ = 116 + \_ + \_$

square them

$16(x^2 + 4x + \cancel{4}) - 9(y^2 - 4y + \cancel{4}) = 116 + 64 + (-36)$

$16(x+2)^2 - 9(y-2)^2 = 144$

$\Downarrow$

divided by 144 on both sides

$\frac{(x+2)^2}{9} - \frac{(y-2)^2}{16} = 1$

a) $\frac{(x-2)^2}{9} - \frac{(y-2)^2}{16} = 1$

b) $\frac{(x+2)^2}{9} - \frac{(y-2)^2}{16} = 1$

c) $\frac{(x+2)^2}{16} + \frac{(y-2)^2}{9} = 1$

d) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

e) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Question 8

Find the x-coordinates of the points of intersection for the functions: $f(x) = x^2 - 6$ and $g(x) = -x + 12$.

$x^2 - 6 = -x + 12$

$\Rightarrow x^2 + x - 18 = 0$

quartic formula $\Rightarrow x = \frac{-1 \pm \sqrt{73}}{2}$

$-\frac{1}{2} + \frac{\sqrt{73}}{2}$ or $-\frac{1}{2} - \frac{\sqrt{73}}{2}$.

a) $\{-1/4 + 1/4\sqrt{73}, 1/2 + 1/2\sqrt{73}\}$

b) $\{-1 - \sqrt{73}, -1 + \sqrt{73}\}$

c) $\{1/2 - 1/2\sqrt{73}, 1/2 + 1/2\sqrt{73}\}$

d) $\{-1/2 - 1/2\sqrt{73}, -1/2 + 1/2\sqrt{73}\}$

e) $\{-13/2 - 1/2\sqrt{73}, -13/2 + 1/2\sqrt{73}\}$

Question 9

Find all roots of the polynomial $P(x) = \frac{3}{4}x^5 - 6x^2$.

find x such that $P(x) = 0 \Rightarrow \frac{3}{4}x^5 - 6x^2 = 0$

$\Rightarrow 3x^5 - 24x^2 = 0$

$\Rightarrow x^5 - 8x^2 = 0$

$\Rightarrow x^2(x^3 - 8) = 0$

$\Rightarrow x = 0$ or 2. $(2^3 = 8)$

a) $\{x = -2, x = -1\}$

b) $\{x = 0, x = 2\}$

c) $\{x = 0, x = 2, x = 3\}$

d) $\{x = -2, x = 0\}$

e) $\{x = 0, x = 3\}$

Question 10

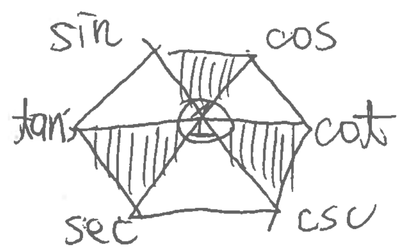
Which of the following are true statements?

✓ I. $\sin^2\theta + \cos^2\theta = 1$

✓ II. $\tan^2\theta + 1 = \sec^2\theta$

✓ III. $1 + \cot^2\theta = \csc^2\theta$

✓ IV. $\frac{1}{\csc^2\theta} + \frac{1}{\sec^2\theta} = 1$



$\Downarrow$

$\sin^2\theta + \cos^2\theta = 1.$

$\frac{1}{\csc\theta} = \sin\theta$

$\frac{1}{\sec\theta} = \cos\theta.$

a) I and III only.

b) II and III only.

c) I, II, and III only.

d) None of these are true.

e) All of these statements are true.

Question 11

Simplify the expression: $\dfrac{7\sec(A)}{\tan(A)+\cot(A)}$ $= 7\dfrac{\frac{1}{\cos(A)}}{\frac{\sin(A)}{\cos(A)}+\frac{\cos(A)}{\sin(A)}}$ ⇒ combine them

$$= 7\frac{\frac{1}{\cos(A)}}{\frac{\sin^2 A+\cos^2 A}{\cos A\sin A}}$$

$$= 7\frac{\frac{1}{\cos A}}{\frac{1}{\cos A\sin A}} = 7\sin A.$$

a) 7 csc(A)

b) 7 sin(A)

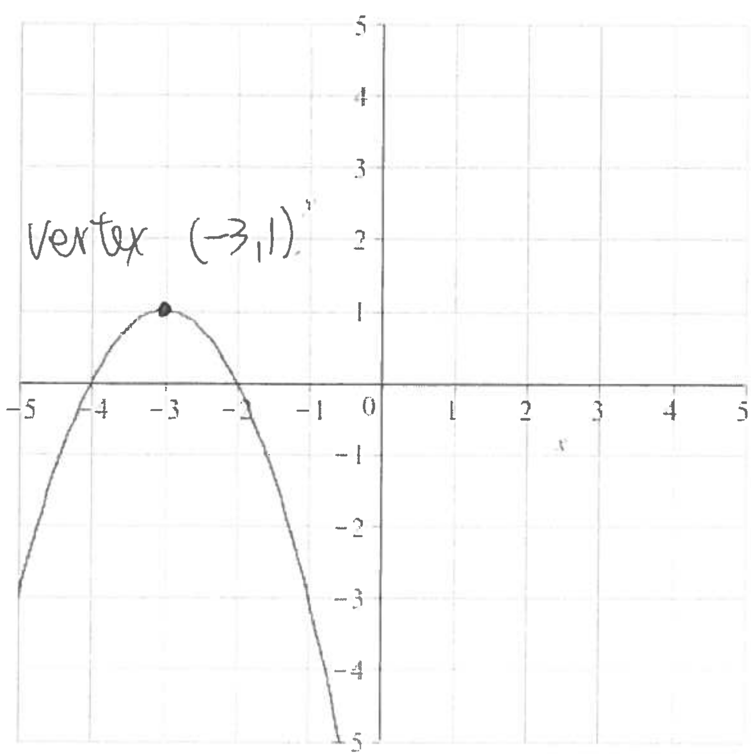
c) 7 sec(A)

d) 7 cot(A)

e) 7 cos(A)

Question 12

Which of the following functions matches the graph below?



a) $f(x) = (x+3)^2 - 1$

b) $f(x) = -(x+3)^2 + 1$

c) $f(x) = -(x-3)^2 + 1$

d) $f(x) = -(x+1)^2 + 3$

e) $f(x) = (x-1)^2 + 3$

Question 13

Given $f(x) = \sqrt{3x-5}$ and $g(x) = x^2 - 4x - 12$, find the domain of $\frac{g}{f}$.

$\Rightarrow f \neq 0$. and the value of f is a real number

$\sqrt{3x-5} \Rightarrow 3x-5>0 \Rightarrow x > \frac{5}{3}$

$\Rightarrow x \in (\frac{5}{3}, \infty)$

a) $[\frac{5}{3}, 6) \cup (6, \infty)$

b) $[\frac{5}{3}, \infty)$

c) $(-\infty, \frac{5}{3}) \cup (\frac{5}{3}, \infty)$

d) $(-\infty, -2) \cup (6, \infty)$

e) $(\frac{5}{3}, \infty)$

Question 14

Perform the indicated operation and reduce completely.

$$\frac{x}{x^2+11x+30} + \frac{3}{x^2+3x-10} - \frac{x}{x^2+4x-12}$$

$$= \frac{x}{(x+5)(x+6)} + \frac{3}{(x+5)(x-2)} - \frac{x}{(x-2)(x+6)}$$

$$= \frac{x(x-2)+3(x+6)-x(x+5)}{(x+6)(x+5)(x-2)}$$

$$= \frac{\cancel{x^2}-2x+3x+18-\cancel{x^2}-5x}{(x+6)(x+5)(x-2)} = \frac{-4x+18}{(x+6)(x+5)(x-2)}$$

a) $\frac{-20x^2 - 18x + 36}{(x+6)(x+5)(x-6)(x-2)}$

b) $\frac{-4x+18}{(x+6)(x+5)(x-2)}$

c) $\frac{x^3 + 10x^2 + 35x + 18}{(x+6)(x+5)(x-2)}$

d) $\dfrac{-x^3 - 12x^2 - 25x + 18}{(x+6)(x+5)(x-2)}$

e) $\dfrac{-22x^2 - 18x + 108}{(x+6)(x+5)(x-6)(x-2)}$

Question 15

Simplify the following:

$$\frac{\left(\frac{x-5}{xy^3}\right)}{\left(\frac{x^2-6x+5}{x^{11}y^{17}}\right)}$$

$$= \frac{x-5}{xy^3} \cdot \frac{x^{11}y^{17}}{x^2-6x+5}$$

$$= \frac{(x-5)}{xy^3} \cdot \frac{x^{11}y^{17}}{(x-5)(x-1)}$$

$$= \frac{x^{10}y^{14}}{x-1}.$$

a) $\dfrac{x+5}{x^{10}y^{20}}$

b) $\dfrac{x-1}{y^{14}x^{10}}$

c) $\dfrac{x-5}{y^{14}x^{12}}$

d) $\dfrac{x^{10}y^{20}}{x+5}$

e) $\dfrac{y^{14}x^{10}}{x-1}$

Question 16

Simplify the following. No answer should contain negative exponents.

$$\frac{x^3y^{-2}z^2}{(3x^{-13}y^5)^{-1}}$$

$$= x^3y^{-2}z^2 \cdot 3x^{-13}y^5$$

$$= x^3\frac{1}{y^2}z^2 \cdot 3\frac{1}{x^{13}}y^5$$

$$= \frac{3x^3z^2y^5}{y^2x^{13}} = \frac{3y^3z^2}{x^{10}}$$

a) $\dfrac{3z^2}{x^{10}y^3}$

b) $\dfrac{-x^{16}z^2}{3y^7}$

c) $\dfrac{-y^3 z^2}{3x^{10}}$

d) $\dfrac{3y^3 z^2}{x^{10}}$

e) $3x^{16}y^3z^2$

Question 17

Given $f(x) = \dfrac{x-1}{x+3}$, simplify $\dfrac{f(x+h)-f(x)}{h}$, $h \neq 0$ when $x = -1$.

$$= \frac{1}{h}\left(\frac{(x+h)-1}{(x+h)+3} - \frac{x-1}{x+3}\right)$$

$x=-1 \Rightarrow$
$$= \frac{1}{h}\left(\frac{h-2}{h+2} - \frac{-2}{2}\right)$$
$$= \frac{1}{h}\left(\frac{h-2}{h+2} + 1\right)$$
$$= \frac{1}{h}\left(\frac{h-2+h+2}{h+2}\right) = \frac{1}{h}\cdot\frac{2h}{h+2} = \frac{2}{h+2}.$$

a) $\dfrac{h-1}{h+3}$

b) $\dfrac{2}{h-2}$

c) 0

d) $\dfrac{2}{h+2}$

e) $h-1$

Question 18

Given that $f(x) = x^2 + 3x$ and $g(x) = 5x - 2$, find $(f \circ g)(2)$.

$g(2) = 5\cdot 2 - 2 = 8$

$= f(g(2))$
$= f(8)$
$= 8^2 + 3\cdot 8$
$= 64 + 24$
$= 88$

a) 48

b) 88

c) 5

d) 24

e) 52

Question 19

Let $f(x) = \dfrac{5x^2-3}{4x^2+5}$. Find the y-intercept of $f(\sqrt{2x+5})$. $= \dfrac{5(2x+5)-3}{4(2x+5)+5} = \dfrac{10x+22}{8x+25}$

$\to x=0 \Rightarrow f(0) = \dfrac{22}{25}$

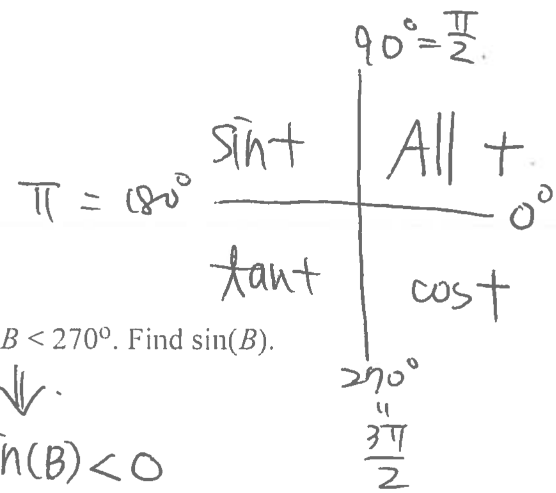
a) $(0, -\frac{3}{5})$

b) $(0, \frac{5}{4})$

c) $(0, \frac{17}{21})$

d) $(0, \frac{22}{25})$

e) $(0, \frac{122}{105})$



Question 20

Suppose that $\sec(B) = -\frac{11}{8}$ and that $180° < B < 270°$. Find $\sin(B)$.

a) $\sin(B) = \frac{\sqrt{57}}{19}$

b) $\sin(B) = -\frac{\sqrt{57}}{11}$

c) $\sin(B) = -\frac{\sqrt{57}}{19}$

d) $\sin(B) = -\frac{\sqrt{3}}{11}$

e) $\sin(B) = \frac{\sqrt{57}}{11}$

① $\sin(B) < 0$

② $\sec(B) = \frac{1}{\cos(B)} \Rightarrow \cos(B) = -\frac{8}{11}$

$\cos^2(B) + \sin^2(B) = 1 \Rightarrow \sin^2(B) = 1 - \frac{64}{121} = \frac{57}{121}$

$\Rightarrow \sin(B) = \pm\frac{\sqrt{57}}{11}$ (but $\sin(B) < 0$)

$\Rightarrow \sin(B) = -\frac{\sqrt{57}}{11}$

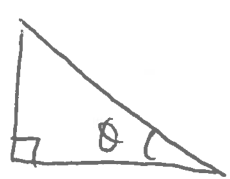
Question 21

<90°

Suppose that θ is an acute angle of a right triangle and that $\sec(\theta) = \frac{8}{5}$. Find $\cos(\theta)$ and $\csc(\theta)$.

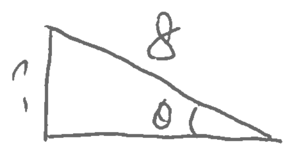
a) $\cos(\theta) = \frac{\sqrt{39}}{8}$ and $\csc(\theta) = \frac{5\sqrt{39}}{39}$

b) $\cos(\theta) = \frac{5}{8}$ and $\csc(\theta) = \frac{8\sqrt{39}}{39}$



and $\sec\theta = \frac{8}{5}$

$\Rightarrow$



$? = \sqrt{64-25} = \sqrt{39}$

$\Rightarrow \cos\theta = \frac{1}{\sec\theta} = \frac{5}{8}$, $\csc\theta = \frac{1}{\sin\theta} = \frac{8}{\sqrt{39}} = \frac{8}{39}\sqrt{39}$

c) $\cos(\theta) = \frac{8}{5}$ and $\csc(\theta) = \frac{8\sqrt{39}}{39}$

d) $\cos(\theta) = \frac{8\sqrt{39}}{39}$ and $\csc(\theta) = \frac{\sqrt{39}}{5}$

e) $\cos(\theta) = \frac{5}{8}$ and $\csc(\theta) = \frac{\sqrt{39}}{8}$

Question 22

List all x-intercepts for $y = -3\sin(\frac{1}{2}x + \frac{\pi}{5})$, on the interval $[-\frac{2\pi}{5}, 4\pi]$.

find x such that $y=0$.

a) $\{\frac{\pi}{5}, \frac{9\pi}{5}, \frac{18\pi}{5}\}$

b) $\{-\frac{2\pi}{5}, \frac{8\pi}{5}, \frac{18\pi}{5}\}$

c) $\{-\frac{2\pi}{5}, \frac{9\pi}{5}, \frac{19\pi}{5}\}$

d) $\{0, \frac{8\pi}{5}, \frac{18\pi}{5}\}$

e) $\{-\frac{2\pi}{5}, \frac{8\pi}{5}, \frac{4\pi}{5}\}$

$\Rightarrow 0 = -3\sin(\frac{x}{2}+\frac{\pi}{5})$

$\Rightarrow 0 = \sin(\frac{x}{2}+\frac{\pi}{5})$.

$\Rightarrow \frac{x}{2}+\frac{\pi}{5} = 0, \pi, 2\pi, 3\pi, \cdots$

$\Rightarrow \frac{x}{2} = -\frac{\pi}{5}, \frac{4\pi}{5}, \frac{9\pi}{5}, \frac{14\pi}{5}$.

$x = -\frac{2}{5}\pi, \frac{8}{5}\pi, \frac{18}{5}\pi, \cancel{\frac{28\pi}{5}}$

Question 23

Solve $\sec^2(x) = 1$ over the interval $[-\frac{\pi}{2}, \frac{5\pi}{2}]$.

$\sec^2 x = 1$

$\Rightarrow \sec^2 x - 1 = 0$

$\Rightarrow \tan^2 x = 0$.

$\Rightarrow \tan x = 0$

$\Rightarrow x = 0, \pi, 2\pi$.

a) $\{-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}\}$

b) $\{0, \pi, 2\pi\}$

c) $\{0, \frac{5\pi}{2}\}$

d) $\{\frac{\pi}{2}, \frac{3\pi}{2}\}$

e) $\{-\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}\}$

Question 24

Given $f(x) = \dfrac{3x^2 - 9x}{2x^2 - 18}$, identify any horizontal asymptotes.

$x \to \infty$

$f = \dfrac{P(x)}{Q(x)}$

$$\begin{cases} \deg P > \deg Q & \lim_{x\to\infty} f \text{ DNE} \\ \deg P = \deg Q & \lim_{x\to\infty} f = \text{leading coeffic} \\ \deg P < \deg Q & \lim_{x\to\infty} f = 0 \end{cases}$$

a) $y = \frac{3}{2}$

$\Rightarrow$ $y = \frac{3}{2}$

b) $y = -3$

c) $y = 3$

d) $y = 0$

e) There are none.

Question 25

Find the exact value of the following expression. If undefined, state, *undefined.*

$$\sin^{-1}(-\frac{\sqrt{3}}{2})$$

$\Rightarrow$ Find x such that $\sin x = -\frac{\sqrt{3}}{2}$

$\Rightarrow x = -\frac{\pi}{3}$

a) $-\frac{\pi}{3}$

b) $\frac{5\pi}{6}$

c) $\frac{\pi}{3}$

d) *undefined*

e) $-\frac{5\pi}{6}$