Math 1431Final Exam Key

1. a. $\frac{7}{9}$ b. $\frac{5}{2}$ c. 3 d. 0 e. 1 f. 0 g. DNE h. 2 i. $\frac{1}{e^2}$ j. $\frac{1}{3}$ k. $\frac{1}{2}$ l. $\frac{3}{4}$

m. $\frac{2}{3}$ n. 1 0. 0 p. $\frac{3}{2}$ q. $\frac{3}{2}$ r. 9 s. 1 t. 1 u. $\frac{1}{2}$

2. $f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h-1}-\sqrt{x+1}}{h} = \cdots = \frac{1}{2\sqrt{x+1}}$

3. a. $A = 4$ b. $A = 6, B = 3$ c. $A = 47, B = 12$

4. a. $A = 2$ b. $A = 3$ c. $Not\ Possible$ d. $A = \frac{9}{4}$

5. a. $f'(x) = \frac{-3}{2}(x^3 - 2x)^{\frac{-3}{2}}(3x^2 - 2)$

b. $f'(x) = -6cos^2(2x)\sin(2x)$

c. $f'(x) = \tan(x) + xsec^2(x)$

d. $f'(x) = -3\cos(2x) + 6xsin(2x)$

e. $f'(x) = \frac{6}{(x+3)^2}$

f. $f'(x) = \cos(x^2 + 2x)(2x + 2)$

g. $f'(x) = \frac{2-x^2}{(x+2)^2}$

h. $f'(x) = (2x - 3)\sin(x^2 - 3x + 5)$

i. $f'(x) = 3(\sin(2x) - \cos(3x))^2(2\cos(2x) + 3\sin(3x))$

j. $f'(x) = 5(4\sin(x) + \cos(5x))^4(4\cos(x) - 5\sin(5x))$

k. $f'(x) = 2\cos(2x)$

l. $f'(x) = -3\sin(3x)$

m. $f'(x) = sec^2(4x)$

n. $f'(x) = -csc^2(x)$

o. $f'(x) = \sec(x)\tan(x)$

p. $f'(x) = -2\csc(2x)\cot(2x)$

q. $f'(x) = -6x^2 + 8x$

r. $f'(x) = \sin(x) + xcos(x)$

s. $f'(x) = \frac{1}{2\sqrt{1+x}}$

t. $f'(x) = \frac{-1}{(x+1)^2}$

u. $f'(x) = 4(\tan(2x) + x)^3(2sec^2(2x) + 1)$

v. $f'(x) = (3x-1)^{\sin(x)}(\cos(x)\ln(3x-1) + sin(x)\left(\frac{3}{3x-1}\right)$

w. $f'(x) = (x+1)^{\ln(x)}\left(\frac{\ln(x)}{x+1} + \frac{\ln(x+1)}{x}\right)$

x. $f'(x) = (x^2+2)^{\frac{1}{\ln(x)}}\left(\frac{-\ln(x^2+2)}{x(\ln(x))^2} + \frac{2x}{\ln(x)(x^2+2)}\right)$

6. $f(-1) < 0 \quad and \quad f(2) > 0$

7. $y - 2 = \frac{2}{13}(x-1)$

8. $y - 6 = -7(x+1)$

9. a. Increasing: $(-\infty, -2) \cup (3, \infty)$

Decreasing: $(-2,3)$

Concave Up: $(-1.5,0) \cup (2, \infty)$

Concave Down: $(-\infty, -1.5) \cup (0,2)$

b. a. Increasing: $(-4,0) \cup (2, \infty)$

Decreasing: $(-\infty, -4) \cup (0,2)$

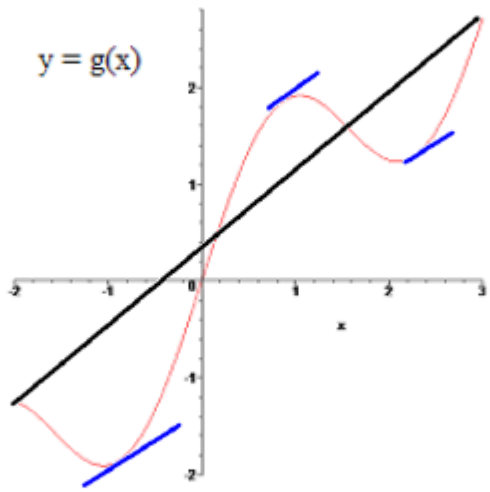
Concave Up: $(-\infty, -2.5) \cup (1, \infty)$

Concave Down: $(-2.5,1)$

10. a. Neither

b. Cannot be determined

11.



12. $\frac{-1}{10}$

13. $\frac{129}{16}$

14. a. $\frac{2}{x}$ b. $\sin\left(x^2\right)$ c. $-3sin((2-3x)^2)$

15. a. $f(x) = (-8x^3 - 6x)(x+2)$ b. $f(x) = -6x^2 + 6x - 1$

16. a. $-2\sqrt{\cot(x)} + C$

b. $\frac{-1}{x} - \frac{1}{x^2} + C$

c. $\frac{3}{4}x^4 - \frac{2}{3}x^3 + 5x + C$

d. $\frac{14}{3}$

e. 4

f. $\frac{1}{12}sin^4(3x) + C$

g. $\frac{1}{3}(51)^{\frac{3}{2}} - \frac{1}{3}(6)^{\frac{3}{2}}$

h. $\frac{1}{3}\sin(x^3 - 6x) + C$

i. $-2\sqrt{9-x^2} + C$

j. $\frac{37}{5184}$

k. $\frac{-1}{2}\cos(2x) + C$

l. $\frac{1}{3}\sin(3x) + C$

m. $\frac{1}{2}\tan(2x) + C$

n. $\frac{-1}{3}\cot(3x) + C$

o. $\frac{1}{2}\sec(2x) + C$

p. $\frac{2}{3}(x+1)^{\frac{3}{2}} + C$

q. $\frac{1}{10}(x^2+1)^5 + C$

r. $\frac{1}{3}\sinh(3x) + \frac{1}{2}\cosh(2x) + C$

s. $\frac{4^{3x}}{3\ln(4)} + C$

t. $\frac{3}{\ln(2)}(\ln(x))^2 + C$

u. $\frac{2^{7x}}{7\ln(2)} - \frac{1}{5}\cosh(5x) + C$

v. $\frac{-1}{12}arctan\left(\frac{\cos(3x)}{4}\right) + C$

w. $\frac{3}{2}arctan\left(\frac{x^2}{2}\right) + C$

x. $\frac{-1}{3}\ln(\cos(3x)) + C$

y. $\frac{1}{6}(\arctan(3x))^2 + C$

17. a. $f'(x) = \lim_{h\to 0}\frac{\sqrt{x+h-1}-\sqrt{x+1}}{h} = \cdots = \frac{1}{2\sqrt{x+1}}$

b. $f'(x) = \lim_{h\to 0}\frac{\frac{2}{x+h-3}-\frac{2}{x-3}}{h} = \cdots = \frac{-2}{(x-3)^2}$

18. a. $y - 2 = -2(x - 5)$

b. $y - 1 = \frac{7}{13}(x - 3)$

19. $\frac{dr}{dt} = \frac{1}{\pi} \frac{in}{sec}$

20. $\frac{dh}{dt} = \frac{3}{20\pi} \frac{ft}{min}$

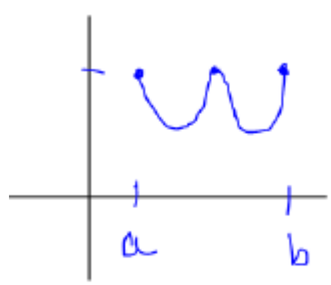
21. $\frac{4}{3}\ in.$

22. $\frac{15}{7} \frac{ft}{sec}$

23. 8

24. a. See Section 3.4

b.



25. Suppose that f is continuous on the closed interval $[a, b]$ and differentiable on the open interval(a, b). There is at least one number c in (a, b) for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

26. a. Domain: $(-\infty, \infty)$

Critical Numbers: $x = \pm 1$

Increasing: $(-\infty, -1) \cup (1, \infty)$

Decreasing: $(-1,1)$

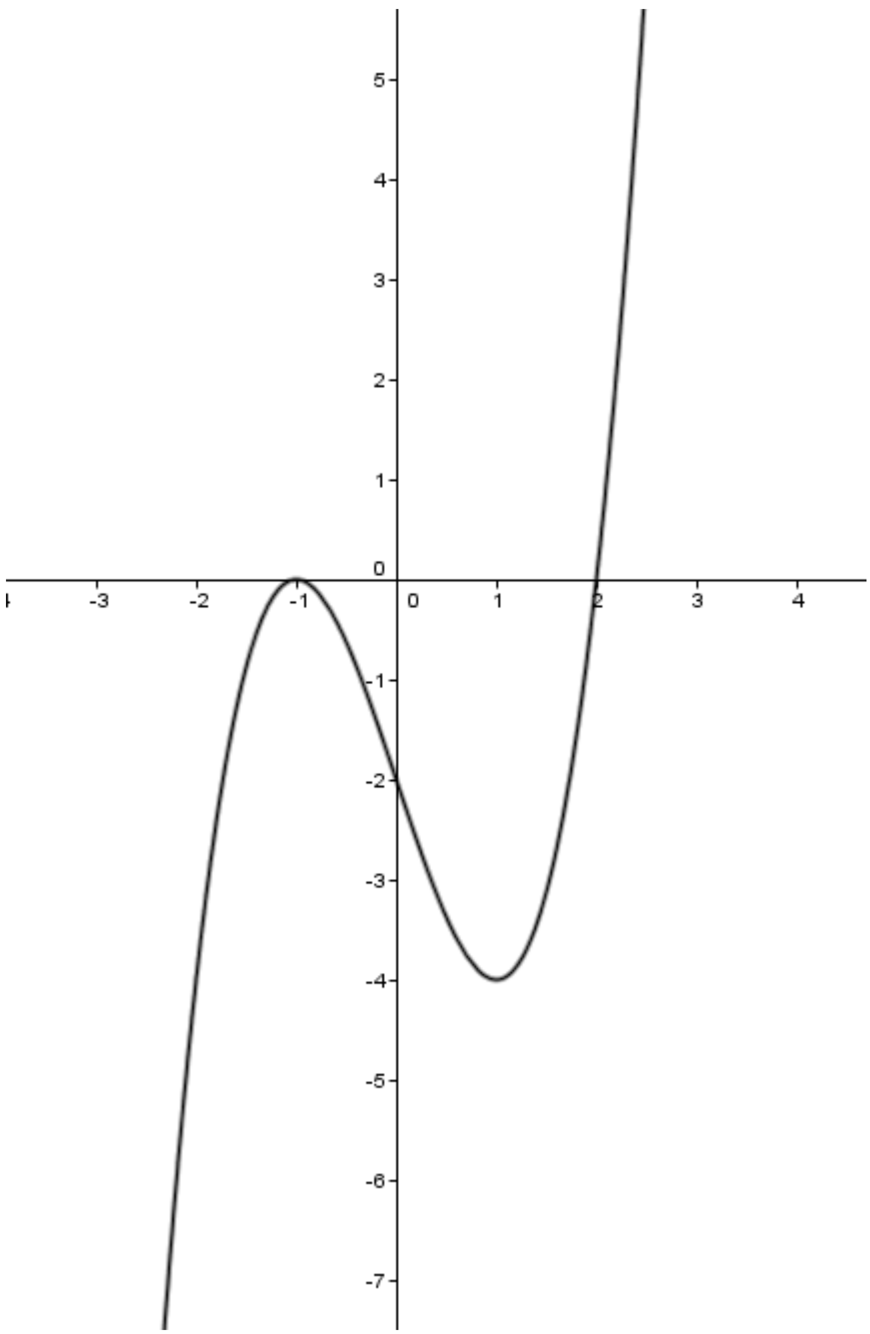
Points of Inflection: $(0, f(0))$

Concave Up: $(0, \infty)$

Concave Down: $(-\infty, 0)$

Local Max: $(-1, f(-1))$

Local Min: $(1, f(1))$



b. Domain: $(-\infty, \infty)$

Critical Numbers: $x = 0, \pm 2$

Increasing: $(-2,0) \cup (2, \infty)$

Decreasing: $(-\infty, -2) \cup (0,2)$

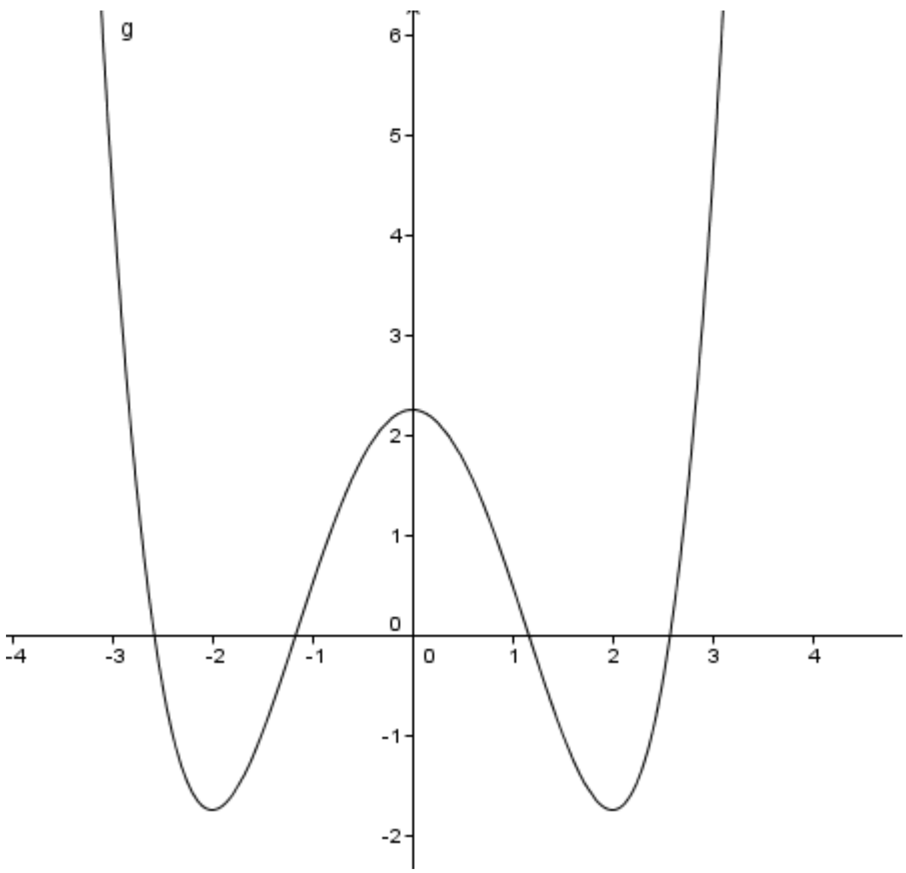
Points of Inflection: $\left(\frac{-2}{\sqrt{3}}, f\left(\frac{-2}{\sqrt{3}}\right)\right), \left(\frac{2}{\sqrt{3}}, f\left(\frac{2}{\sqrt{3}}\right)\right)$

Concave Up: $\left(-\infty, \frac{-2}{\sqrt{3}},\right) \cup \left(\frac{2}{\sqrt{3}}, \infty\right)$

Concave Down: $\left(\frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}},\right)$

Local Max: $(0, f(0))$

Local Min: $\left(-2, f(-2)\right), (2, f(2))$



27. $c = \pm\frac{\sqrt{3}}{3}$

28. $\frac{90\sqrt{3}+\pi}{180}$

29. $\frac{23}{4}$