PRINTABLE VERSION

Quiz 9

Sol

Question 1

Differentiate the function: $f(x) = (x^2 - 3)^4$

a) $f'(x) = 8x(x^2 - 3)^3$

b) $f'(x) = 4(x^2 - 3)^3$

c) $f'(x) = 4x(2x)^3$

d) $f'(x) = 8(x^2 - 3)^3$

e) $f'(x) = 2x(x^2 - 3)^3$

$f'(x) = 4 \cdot 2x(x^2-3)^3$

$= 8x(x^2-3)^3$

Question 2

Calculate the derivative of the given function: $f(x) = 10x^3\cot(x)$

a) $f'(x) = 30x^2\csc(x) - 10x^3\csc(x)\cot(x)$

b) $f'(x) = 30x^2\csc(x)\cot(x)$

c) $f'(x) = -30x^2\csc^2(x)$

d) $f'(x) = 30x^2\csc^2(x)$

e) $f'(x) = 30x^2\cot(x) - 10x^3\csc^2(x)$

$f'(x) = 30x^2\cot(x)$

$+10x^3(-\csc^2(x))$

$= 30x^2\cot(x) - 10x^3\csc^2(x)$.

Question 3

Determine the value(s) of x for which $f'(x) = 0$ given that $f(x) = (-x^2 + 9)^2$

a) $x = \frac{1}{3}$ and $x = -\frac{1}{3}$

b) $x = 0$

c) $x = 0$, $x = \frac{1}{3}$ and $x = -\frac{1}{3}$

d) $x = 0$, $x = 3$ and $x = -3$

$f'(x) = 2 \cdot (-2x)(-x^2+9)$.

$= 2 \cdot (2x)(x+3)(x-3) = 0$

$x = 0$ or 3 or -3

e) $x = 3$ and $x = -3$

Question 4

Determine the value(s) of x for which $f'(x) > 0$ given that $f(x) = (-16x^2 + 36)^2$

a) $\left(-\frac{3}{2}, \frac{3}{2}\right)$

b) $\left(-\infty, -\frac{3}{2}\right) \cup \left(0, \frac{3}{2}\right)$

c) $\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$

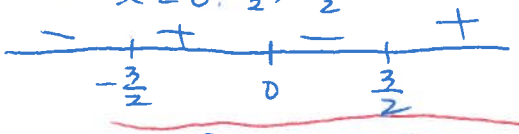
d) $\left(-\frac{3}{2}, 0\right) \cup \left(0, \frac{3}{2}\right)$

e) $\left(-\frac{3}{2}, 0\right) \cup \left(\frac{3}{2}, \infty\right)$

$f'(x) = 2 \cdot (-32x)(-16x^2+36)$.

$= 2 \cdot (+32x)(4x+6)(4x-6) > 0$

$x = 0, \frac{3}{2}, -\frac{3}{2}$



$\Leftrightarrow x \in (-\frac{3}{2}, 0) \cup (\frac{3}{2}, \infty)$.

Question 5

Find $\frac{dy}{dx}$ given $y = \sqrt{6x^5 + 11}$

a) $\frac{dy}{dx} = \frac{-30x^4}{\sqrt{6x^5 + 11}}$

b) $\frac{dy}{dx} = \frac{15x^4}{\sqrt{6x^5 + 11}}$

c) $\frac{dy}{dx} = 30x^4\sqrt{6x^5 + 11}$

d) $\frac{dy}{dx} = 15x^4\sqrt{6x^5 + 11}$

e) $\frac{dy}{dx} = -\frac{30x^4}{\sqrt{6x^5 + 11}}$

$\frac{dy}{dx} = \frac{1}{2}(6x^5+11)^{-\frac{1}{2}} \cdot 30x^4$

$\frac{1}{(6x^5+11)^{\frac{1}{2}}}$

$= \frac{15x^4}{\sqrt{6x^5+11}}$

Question 6

Find $\frac{dy}{dx}$ given $5x^3 + 5y^3 - 2xy = 0$

a) $\frac{dy}{dx} = -\frac{5x + y}{x + 5y}$

$\frac{d}{dx}(5x^3+5y^3-2xy) = \frac{d0}{dx} = 0$

$\Rightarrow 15x^2 + 5 \cdot 3y^2\frac{dy}{dx} - 2y - 2x\frac{dy}{dx} = 0$

$\Rightarrow (15y^2 - 2x)\frac{dy}{dx} = 2y - 15x^2$.

$\frac{dy}{dx} = \frac{2y - 15x^2}{15y^2 - 2x}$

b) $\frac{dy}{dx} = \frac{15x^2 + 2y}{15y^2 - 2x}$

c) $\frac{dy}{dx} = \frac{-15x^2 + 2y}{15y^2 - 2x}$

d) $\frac{dy}{dx} = \frac{-5x - y}{-x + 5y}$

e) $\frac{dy}{dx} = \frac{-1x + 5y}{4y - 5x}$

Question 7

Find $\frac{dy}{dx}$ given $(5x - y)^2 - 2y = 2$.

a) $\frac{dy}{dx} = \frac{25x - 5y}{-y - 5x - 1}$

b) $\frac{dy}{dx} = \frac{-25x + 5y}{-y + 5x + 1}$

c) $\frac{dy}{dx} = \frac{2x - 10y}{-25y + 5x - 1}$

d) $\frac{dy}{dx} = \frac{x - 5y}{-25y - 5x - 1}$

e) $\frac{dy}{dx} = \frac{25x - 5y}{-y - 5x + 1}$

$\frac{d}{dx}[(5x-y)^2 - 2y] = \frac{d}{dx}(2) = 0$

$2(5x-y)\cdot\left[5 - \frac{dy}{dx}\right] - 2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{25x - 5y}{5x - y + 1}$

Question 8

Find $\frac{dy}{dx}$ given $\tan(xy) = 5x + 2y$.

a) $\frac{dy}{dx} = \frac{5 + y\sec^2(xy)}{x\sec^2(xy) - 2y}$

b) $\frac{dy}{dx} = \frac{5 - y\sec^2(xy)}{x\sec^2(xy) - 2}$

c) $\frac{dy}{dx} = \frac{5 - y\sec^2(xy)}{x\sec^2(xy) + 2}$

d) $\frac{dy}{dx} = \frac{5 - y\sec^2(xy)}{x\sec(xy)\tan(xy) - 2}$

$\frac{d}{dx}(\tan(xy)) = \frac{d}{dx}(5x + 2y)$

$\sec^2(xy)\cdot\left[y + x\frac{dy}{dx}\right] = 5 + 2\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{y\sec^2(xy) - 5}{2 - x\sec^2(xy)}$

e) $\frac{dy}{dx} = \frac{5 - y\sec(xy)\tan(xy)}{x\sec(xy)\tan(xy) - 2}$

Question 9

Find $\frac{d^2y}{dx^2}$ in terms of x and y given $xy + 2y^2 = 6$.

a) $\frac{d^2y}{dx^2} = \frac{-y}{4y + x}$

b) $\frac{d^2y}{dx^2} = \frac{-y - 6}{4y - x}$

c) $\frac{d^2y}{dx^2} = \frac{-y + 4x - 6}{x}$

d) $\frac{d^2y}{dx^2} = \frac{2xy - 4y^2}{(4y + x)^3}$

e) $\frac{d^2y}{dx^2} = \frac{2xy + 4y^2 - 12x - 144}{(4y + x)^3}$

$\frac{d}{dx}(xy + 2y^2) = \frac{d}{dx}(6) = 0$

$\Rightarrow y + x\frac{dy}{dx} + 4y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x+4y}$ —(*)

again $\Rightarrow \frac{d}{dx}\left(y + x\frac{dy}{dx} + 4y\frac{dy}{dx}\right) = 0$

$\Rightarrow \frac{dy}{dx} + \frac{dy}{dx} + x\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\cdot\frac{dy}{dx} + 4y\frac{d^2y}{dx^2} = 0$

by (*) $\Rightarrow -\frac{2y}{x+4y} + 4\cdot\left(\frac{y^2}{(x+4y)^2}\right) + (x+4y)\frac{d^2y}{dx^2} = 0$

$\frac{d^2y}{dx^2} = \frac{2y}{(x+4y)^2} - 4\frac{y^2}{(x+4y)^3} = \frac{2xy + 4y^2}{(x+4y)^3}$

Question 10

Compute $\frac{d^2y}{dx^2}$ at the point $(3, 2)$ given $x^2 - y^2 = 5$.

a) $\frac{5}{8}$

b) $-\frac{5}{4}$

c) $-\frac{15}{8}$

d) $-\frac{5}{8}$

e) $\frac{5}{4}$

$\frac{d}{dx}(x^2 - y^2) = \frac{d}{dx}5 = 0 \Rightarrow 2x - 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{x}{y}$ (*)

do "$\frac{d}{dx}$" $\frac{d}{dx}\left(2x - 2y\frac{dy}{dx}\right) = 0 \Rightarrow 2 - 2\frac{dy}{dx}\frac{dy}{dx} - 2y\frac{d^2y}{dx^2} = 0$

by (*) $\Rightarrow 2 - 2\left(\frac{x}{y}\right)\left(\frac{x}{y}\right) - 2y\frac{d^2y}{dx^2} = 0$

$(3,2) \Rightarrow 2 - 2\left(\frac{3}{2}\right)\left(\frac{3}{2}\right) - 2\cdot 2\frac{d^2y}{dx^2} = 0 \Rightarrow \frac{d^2y}{dx^2} = -\frac{5}{8}$

Question 11

Find the equation of the tangent line to the curve at the point $\left(-\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$ given $x - \cos(5y) = 0$.

a) $y=\frac{-2\sqrt{2}}{5}\left(x-\frac{\sqrt{2}}{2}\right)-\frac{\pi}{4}$

b) $y=-\frac{\sqrt{2}}{5}\left(x+\frac{\sqrt{2}}{2}\right)+\frac{\pi}{4}$

c) $y=\frac{5\sqrt{2}}{2}\left(x+\frac{\sqrt{2}}{2}\right)-\frac{\pi}{4}$

d) $y=\frac{\sqrt{2}}{5}\left(x-\frac{\sqrt{2}}{2}\right)-\frac{\pi}{4}$

e) $y=\frac{-5\sqrt{2}}{2}\left(x+\frac{\sqrt{2}}{2}\right)-\frac{\pi}{4}$

Find slope $\frac{dy}{dx}$: $\frac{d}{dx}(x-\cos(5y))=0$

$\Rightarrow 1-(-\sin(5y))\cdot 5\frac{dy}{dx}=0$

$\frac{dy}{dx}=-\frac{1}{5\sin(5y)}$

at $(-\frac{\sqrt{2}}{2},\frac{\pi}{4})\Rightarrow \left.\frac{dy}{dx}\right|_{(-\frac{\sqrt{2}}{2},\frac{\pi}{4})}=\frac{-1}{5\cdot\sin(5\frac{\pi}{4})}=\frac{-1}{5\cdot(-\frac{\sqrt{2}}{2})}=\frac{2}{5\sqrt{2}}=\frac{\sqrt{2}}{5}$

$\Rightarrow$ line $y-\frac{\pi}{4}=\frac{\sqrt{2}}{5}(x+\frac{\sqrt{2}}{2})$

Question 12

Find $\frac{dy}{dx}$ given $\frac{5x}{\sqrt{x^2+5}}=y$ (I Guess!)

a) $\frac{dy}{dx}=\frac{25}{\sqrt{x^2+5}}$

b) $\frac{dy}{dx}=\frac{25}{(x^2+5)^{3/2}}$

c) $\frac{dy}{dx}=\frac{25x}{(x^2+5)^{3/2}}$

d) $\frac{dy}{dx}=\frac{25x^2}{\sqrt{x^2+5}}$

e) $\frac{dy}{dx}=\frac{-25}{(x^2+5)^{3/2}}$

$\frac{dy}{dx}=y'=\left(\frac{5x}{\sqrt{x^2+5}}\right)'=\left(5x\cdot[(x^2+5)^{-\frac{1}{2}}]\right)'$

$=5\cdot(x^2+5)^{-\frac{1}{2}}+5x\cdot(-\frac{1}{2})(x^2+5)^{-\frac{3}{2}}\cdot 2x$

↑ product rule

$=\frac{5(x^2+5)-5x^2}{(x^2+5)^{\frac{3}{2}}}=\frac{25}{(x^2+5)^{\frac{3}{2}}}$