PRINTABLE VERSION Sol.

Quiz 8

Question 1

Find the derivative of the function $G(x) = (4x^3 + 2x^2)^5$.

a) $G'(x) = 5(-4x^3 + 2x^2)^4$

b) $G'(x) = 5(12x^2 + 4x)^4$

c) $G'(x) = 5(4x^3 + 2x^2)^4(12x^2 + 4x)$

d) $G'(x) = (4x^3 + 2x^2)^4(12x^2 + 4x)$

e) $G'(x)$ does not exist.

$$G'(x) = 5(4x^3+2x^2)^4 \cdot [4x^3+2x^2]'$$

chain rule

$$= 5(4x^3+2x^2)^4 \cdot [12x^2+4x]$$

Question 2

Find the derivative of the function $f(x) = 4x^2\cos(x) - x$.

a) $f'(x) = 8x\cos(x) - 4x^2\sin(x)$

b) $f'(x) = 8x\cos(x) - 4x^2\sin(x) - 1$

c) $f'(x) = -8x\cos(x) + 4x^2\sin(x) - 1$

d) $f'(x)$ does not exist.

e) $f'(x) = -8x\sin(x) - 1$

$$f'(x) = [4x^2]'\cos(x) + 4x^2[\cos(x)]' - 1$$

product rule

$$= \cancel{16x}\ 8x \cdot \cos(x) + 4x^2[-\sin(x)] - 1$$

$$= 8x\cos(x) - 4x^2\sin(x) - 1$$

Question 3

Find $\frac{d^2}{dx^2}\left[\left(5x^2 - 4x\right)\cos(x)\right]$.

a) $(-5x^2 + 4x + 10)\sin(x) - (20x - 8)\cos(x)$

b) Does not exist.

c) $(-5x^2 + 4x + 10)\cos^2(x) - (20x - 8)\sin(x)$

d) $(10x - 4)\cos(x) - (5x^2 - 4x)\sin(x)$

e) $(-5x^2 + 4x + 10)\cos(x) - (20x - 8)\sin(x)$

$$= \frac{d}{dx}\left\{\frac{d}{dx}[(5x^2-4x)\cos(x)]\right\}$$

$$= \frac{d}{dx}\left\{(5x^2-4x)'\cos(x) + (5x^2-4x)[\cos(x)]'\right\}$$

product rule

$$= \frac{d}{dx}\left\{(10x-4)\cos(x) - (5x^2-4x)\sin(x)\right\}$$

$$= (10x-4)'\cos(x) + (10x-4)[\cos(x)]' - \left\{(5x^2-4x)'\sin(x) + (5x^2-4x)[\sin(x)]'\right\}$$

product

$$= 10\cos(x) - (10x-4)\sin(x)$$

$$- (10x-4)\sin x - (5x^2-4x)\cos(x)$$

$$= (-5x^2+4x+10)\cos(x) - (20x-8)\sin(x)$$

Question 4

For $g(x) = \frac{1}{4x^2 + 5x}$, find $g'(2)$.

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a) $\frac{1}{676}$

b) $\frac{676}{21}$

c) $\frac{21}{676}$

d) $-\frac{21}{676}$

e) $-\frac{1}{676}$

$$g'(x) = \left(\frac{1}{4x^2+5x}\right)' = \left[(4x^2+5x)^{-1}\right]'$$

$$= -1 \cdot (4x^2+5x)^{-2} \cdot [4x^2+5x]'$$

chain rule

$$= -(4x^2+5x)^{-2}(8x+5)$$

$$= -\frac{8x+5}{(4x^2+5x)^2}$$

$$g'(2) = -\frac{8\cdot 2+5}{(4\cdot 2^2+5\cdot 2)^2} = -\frac{21}{(26)^2} = \boxed{-\frac{21}{676}}$$

Question 5

$\frac{d}{dx}\left(\left(x^2 - 2x\right)\cdot\frac{d}{dx}\left(x + \frac{6}{x}\right)\right)$ =

a) $3x^2 - 4x + 6$

b) $2x - 2 - \frac{12}{x^2}$

c) $2x^2 + 12 - 2x - \frac{12}{x}$

d) $6x - 4$

e) $2 + \frac{24}{x^3}$

$x + 6\cdot x^{-1}$

$$= \frac{d}{dx}\left((x^2-2x)\cdot(1-6\cdot x^{-2})\right).$$

$$= (2x-2)(1-6x^{-2}) + (x^2-2x)(+12x^{-3})$$

product

$$= (2x-2)\left(1-\frac{6}{x^2}\right) + \frac{(x^2-2x)12}{x^3}$$

$$= 2x - 2 - \cancel{\frac{12}{x}} + \frac{12}{x^2} + \cancel{\frac{12}{x}} - \frac{24}{x^2}$$

$$\boxed{= 2x - 2 - \frac{12}{x^2}}$$

Question 6

Find $\frac{dy}{dx}$ at $x = 0$ given $y = u + \frac{1}{u}$ and $u = (1x + 1)^5$.

a) $\frac{dy}{dx} = 0$

b) $\frac{dy}{dx} = -1$

c) $\frac{dy}{dx} = 1$

d) $\frac{dy}{dx} = 2$

e) $\frac{dy}{dx} = 3$

Here $y = y(u(x))$

$$\frac{dy}{du} = 1 - u^{-2} = 1 - \frac{1}{u^2}, \quad \frac{du}{dx} = 5(x+1)^4 \cdot 1$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$= \left[1 - \frac{1}{((x+1)^5)^2}\right]\cdot\left[5(x+1)^4\right].$$

at $x=0$, we have $\left.\frac{dy}{dx}\right|_{x=0} = \left[1 - \frac{1}{((0+1)^5)^2}\right]\cdot\left[5\cdot 1^4\right]$

$$= [1-1]\cdot[5] = 0$$

Question 7

Evaluate $(g \circ f)'(9)$, given that:

$f(8) = 8$	$f'(8) = 8$
$f(9) = 9$	$f'(9) = 8$
$f(10) = 9$	$f'(10) = 9$
$g(8) = 8$	$g'(8) = 9$
$g(9) = 10$	$g'(9) = 10$
$g(10) = 9$	$g'(10) = 10$

① ② ③

$$(g\circ f)'(9) = [g(f(x))]'\Big|_{x=9}$$

$$\underset{\uparrow \text{chain rule}}{=} [g'(f(x))\cdot f'(x)]\Big|_{x=9} = g'(f(9))\cdot f'(9) \underset{\text{by ①②}}{=} g'(9)\cdot 8 \underset{\text{by ③}}{=} 10\cdot 8 = \boxed{80}$$

a) 79

b) 82

c) 81

d) 80

e) 83

Question 8

Express the deriviative $\frac{d}{dx}\left((f(2x))^2 - 1\right)$ in terms of f'.

a) $f(2x)\cdot f'(2x)$

b) $2\cdot f(2x)\cdot f'(2x)$

c) $4\cdot f(2x)\cdot f'(2x)$

d) $4x\cdot f'(2x)$

e) $4\cdot f'(2x)$

$$\overset{\text{chain rule}}{=} 2(f(2x))\cdot f'(2x)\cdot(2x)'$$

$$= 2\cdot(f(2x))\cdot f'(2x)\cdot 2$$

$$= \boxed{4\cdot f(2x)\cdot f'(2x)}$$

Question 9

Calculate the derivative of the given function $f(x) = 4\sin^5(\sqrt{x})$ $= 4\cdot(\sin(\sqrt{x}))^5$

a) $f'(x) = 20\cos(\sqrt{x})$

b) $f'(x) = \frac{40\sin^4(\sqrt{x})\cos(\sqrt{x})}{\sqrt{x}}$

c) $f'(x) = \frac{10\sin^4(\sqrt{x})\cos(\sqrt{x})}{\sqrt{x}}$

d) $f'(x) = 20\sin^4(\sqrt{x})\cos(\sqrt{x})$

e) $f'(x) = \frac{10\cos(\sqrt{x})}{\sqrt{x}}$

$$f'(x) \underset{\uparrow \text{chain rule}}{=} 4\cdot 5(\sin(\sqrt{x}))^4\cdot[\sin(\sqrt{x})]'$$

$$= 20(\sin(\sqrt{x}))^4\cdot\cos(\sqrt{x})\cdot(\sqrt{x})'$$

$$= 20\cdot(\sin(\sqrt{x}))^4\cdot\cos(\sqrt{x})\cdot\frac{1}{2}x^{-\frac{1}{2}}$$

$$= \boxed{\frac{10\sin^4(\sqrt{x})\cos(\sqrt{x})}{\sqrt{x}}}$$

Question 10

Find the equation of the tangent line for $f(x) = 4\tan(x)$ at $x = \frac{\pi}{4}$

Slope of tangent line at $x = \frac{\pi}{4}$ or point $(\frac{\pi}{4}, f(\frac{\pi}{4})) = (\frac{\pi}{4}, 4)$

is $f'(\frac{\pi}{4}) \underset{\uparrow}{=} 4\cdot\sec^2(\frac{\pi}{4}) = 4\cdot(\frac{2}{\sqrt{2}})^2 = 8$.

a) $y = 8\left(x - \frac{\pi}{4}\right) + 4$

b) $y = \left(x - \frac{\pi}{4}\right) + 4\sqrt{2}$

c) $y = 8\left(x - \frac{\pi}{4}\right)$

d) $y = 4\left(x - \frac{\pi}{4}\right) + 8$

e) $y = 4\left(x - \frac{\pi}{4}\right) + 4\sqrt{2}$

$f'(x) = 4\sec^2(x)$

line: $\boxed{y - 4 = 8\cdot(x - \frac{\pi}{4})}$.

Question 11

Determine the value(s) of x between 0 and 2π where the tangent lines are horizontal for $f(x) = 10\sin(x) - 10\cos(x)$.

a) $x = \frac{3\pi}{4}$ and $x = \frac{5\pi}{4}$

b) $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$

c) $x = 0$ and $x = \pi$

d) $x = \frac{3\pi}{4}$ and $x = \frac{7\pi}{4}$

e) $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$

$\Leftrightarrow$ Find $x \in (0, 2\pi)$ such that $f'(x) = 0$.

$f'(x) = 10\cos(x) - 10\cdot(-\sin(x))$

$= 10\cos(x) + 10\sin(x)$ which is 0

$\Rightarrow 10\cos(x) = -10\sin(x)$

$\Rightarrow \cos(x) = -\sin(x)$

$\Rightarrow \boxed{x = \frac{3\pi}{4} \text{ or } \frac{7\pi}{4}}$.

sin +	All +
tan +	cos +