Sol

PRINTABLE VERSION

Quiz 7

Question 1

Find the derivative of $f(x) = 5x^2 + 2x + 1$.

a) $f'(x) = 2x - 2$

b) $f'(x) = 5x + 2$

c) $f'(x) = 10x + 2 + \frac{1}{x}$

d) $f'(x) = 10x + 2$

e) $f'(x) = 10x + 3$

$f'(x) = 10x + 2$

Question 2

Find the derivative of $f(x) = \frac{7}{x^2} - 7x^3$.

a) $f'(x) = \frac{14}{x^3} - 21x^2$

b) $f'(x) = -21x^2 + 14x$

c) $f'(x) = -\frac{14}{x^3} - 21x^2$

d) $f'(x) = -\frac{7}{x^3} + 21x^2$

e) $f'(x) = \frac{7}{x^3} - 21x^2$

$f(x) = 7 \cdot x^{-2} - 7x^3$

$f'(x) = -14x^{-3} - 21x^2$

$= \frac{-14}{x^3} - 21x^2$

Question 3

Find the slope of the line that is tangent to the graph of $f(x) = x^6 + 5x^5 - x^2 + 1$ at $x = 1$.

a) 23

b) 19

c) 21

d) 16

e) 17

slope: $f'(x) = 6x^5 + 15x^2 - 2x$

$\Rightarrow f'(1) = 6 + 15 - 2 = 19$

Question 4

Consider the function $f(x) = x^3 + 4x^2 + 3$. Find the equation of the normal line at the point $(1, 8)$.

a) $y = -11x + 19$

b) $y = \frac{x}{11} + \frac{89}{11}$

$f'(x) = 3x^2 + 8x$

slope of tangent line at $x = 1$, $\Rightarrow f'(1) = 11$

slope of normal $= \frac{-1}{\text{slope of tangent}} = -\frac{1}{11}$

Normal line: $(y - 8) = -\frac{1}{11}(x - 1)$

c) $y = \frac{x}{11} + \frac{87}{11}$

d) $y = \frac{x}{11} - \frac{87}{11}$

e) $y = 11x - 3$

Question 5

Consider the function $f(x) = x^4 - x^2 + 4$. Find the points where the tangent line is horizontal.

a) $\left(\frac{1}{2}, \frac{61}{16}\right)\left(-\frac{1}{2}, \frac{61}{16}\right)$

b) $(-2, 16)$

c) $(-2, 16)(2, 16)$

d) $(0, 4)\left(-\frac{\sqrt{2}}{2}, \frac{15}{4}\right)\left(\frac{\sqrt{2}}{2}, \frac{15}{4}\right)$

e) $(0, 0)\left(-\frac{\sqrt{2}}{2}, 0\right)\left(\frac{\sqrt{2}}{2}, 0\right)$

Find x such that $\Leftrightarrow f'(x) = 0$

$f'(x) = 4x^3 - 2x \Rightarrow 4x^3 - 2x = 0$

$\Rightarrow 2x(2x^2 - 1) = 0$

$\Rightarrow 2x(\sqrt{2}x + 1)(\sqrt{2}x - 1) = 0$

$x = 0, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \Rightarrow (0, f(0))$

$\left(\frac{1}{\sqrt{2}}, f\left(\frac{1}{\sqrt{2}}\right)\right)$

$\left(-\frac{1}{\sqrt{2}}, f\left(\frac{-1}{\sqrt{2}}\right)\right)$

Question 6

Given the function $f(x) = \frac{1}{3}x^3 - 2x^2 + 7x + 1$. Find the points where the tangent line has slope 4.

a) $(-3, -47), \left(1, \frac{19}{3}\right)$

b) $(3, 13), \left(1, \frac{19}{3}\right)$

c) $(-3, -47), \left(-1, -\frac{25}{3}\right)$

d) $(0, 1), \left(-1, -\frac{25}{3}\right)$

e) $(0, 1), (3, 13)$

$\Leftrightarrow f(x) = 4$

$f(x) = x^2 - 4x + 7 = 4$

$\Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow (x - 3)(x - 1) = 0$

$\Rightarrow x = 1$ or 3

points: $(1, f(1))$, $(3, f(3))$

Question 7

Let $f(x) = 2\cos(x)$. find $f'\left(\frac{\pi}{2}\right)$.

a) -4

b) -2

c) -1

d) 0

e) 2

$f'(x) = -2\sin(x)$

$f'\left(\frac{\pi}{2}\right) = -2\sin\left(\frac{\pi}{2}\right) = -2$

Question 8

For $g(x) = x + 5\sin(x) + \cot(x)$, find $g'\left(\frac{\pi}{4}\right)$.

a) $2 + \sqrt{3}$

b) $\frac{5\sqrt{2}}{2} - 1$

c) $2 - \sqrt{3}$

d) $\frac{5\sqrt{2}}{2} - 1 + \frac{\pi}{4}$

e) $\frac{\sqrt{3}}{2} + \frac{\pi}{4}$

$g'(x) = 1 + 5\cos x - \csc^2 x$

$g'\left(\frac{\pi}{4}\right) = 1 + 5\cos\frac{\pi}{4} - \csc^2\left(\frac{\pi}{4}\right) = 1 + \frac{5\sqrt{2}}{2} - 2 = \frac{5\sqrt{2}}{2} - 1$

Question 9

Determine the number(s), x, between 0 and 2π where the line tangent to the function $f(x) = 6\sin(x) + 6\cos(x)$ is horizontal.

a) $x = \left\{\frac{\pi}{8}, \frac{5\pi}{4}\right\}$

b) $x = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

c) $x = \left\{\frac{\pi}{4}, \frac{5\pi}{2}\right\}$

d) $x = \{0, 1\}$

e) $x = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

$\Leftrightarrow$ Find x such that $f'(x) = 0$: $x \in (0, 2\pi)$.

$f'(x) = 6\cos(x) - 6\sin(x) = 0$

$\Rightarrow 6\cos(x) = 6\sin(x) \Rightarrow \cos(x) = \sin(x)$. $x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$.

Question 10

Find the third derivative of the function $f(x) = 3x^3 - \frac{8}{x^3}$

a) $f'''(x) = 18x + \frac{96}{x^5}$

b) $f'''(x) = 9x^2 + \frac{24}{x^4}$

c) $f'''(x) = 18 + \frac{480}{x^6}$

d) $f'''(x) = 18x - \frac{96}{x^5}$

e) $f'''(x) = 18 - \frac{480}{x^6}$

Find $f^{(3)}(x)$ as $f(x) = 3x^3 - 8x^{-3}$

$f'(x) = 9x^2 + 24x^{-4}$

$f''(x) = 18x - 96x^{-5}$

$f^{(3)}(x) = 18 + 480x^{-6}$

$= 18 + \frac{480}{x^6}$