PRINTABLE VERSION

Quiz 6 Sol.

Question 1

D

Given $f(x) = \dfrac{7}{\sqrt{x+2}}$ which of the following expressions will represent $f'(x)$? $\overset{\text{def}}{=} \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{\frac{7}{\sqrt{x+h+2}} - \frac{7}{\sqrt{x+2}}}{h}$

a) $\lim\limits_{h\to 0} \dfrac{\frac{7}{\sqrt{x+h+2}}}{h}$

b) $\lim\limits_{h\to x} \dfrac{\left(\frac{7}{\sqrt{x+h+2}}\right) - \left(\frac{7}{\sqrt{x+2}}\right)}{h}$

c) $\dfrac{\left(\frac{7}{\sqrt{x+h+2}}\right) - \left(\frac{7}{\sqrt{x+2}}\right)}{h}$

d) $\lim\limits_{h\to 0} \dfrac{\left(\frac{7}{\sqrt{x+h+2}}\right) - \left(\frac{7}{\sqrt{x+2}}\right)}{h}$

e) $\lim\limits_{h\to 0} \dfrac{\left(\frac{7}{\sqrt{x+2}} + h\right) - \left(\frac{7}{\sqrt{x+2}}\right)}{h}$

Question 2

C

$\lim\limits_{h\to 0} \dfrac{\frac{1}{6+h} - \frac{1}{6}}{h} = \lim\limits_{h\to 0} \dfrac{\frac{6-(6+h)}{6(6+h)}}{h} = \lim\limits_{h\to 0} \dfrac{-h}{6(6+h)} \cdot \dfrac{1}{h}$

$= \lim\limits_{h\to 0} \dfrac{-1}{6(6+h)} = \dfrac{-1}{36}$

a) does not exist

b) $\dfrac{1}{6}$

c) $-\dfrac{1}{36}$

d) 0

e) $-\dfrac{1}{6}$

Question 3

B

The limit $\lim\limits_{h\to 0} \dfrac{(2+h)^2 - 4}{h}$ represents the derivative of a function f at a number c. Determine f and c.

Compare this with $\lim\limits_{h\to 0} \dfrac{f(c+h)-f(c)}{h}$

a) $f(x) = (2+x)^2, c = -2$

b) $f(x) = x^2, c = 2$

c) $f(x) = (2+x)^2, c = 2$

d) $f(x) = (2-x)^2, c = 4$

e) $f(x) = x^2, c = 4$

We have $f(c+h) = (2+h)^2$ and $f(c) = 4$.

$\Rightarrow f(x) = x^2$ and $c = 2$

Question 4

$$\cos\frac{\pi}{6}\sin(h)-\sin\frac{\pi}{6}\cos(h)=\frac{\sqrt{3}}{2}\sin(h)-\frac{1}{2}\cos(h)$$

B

The limit $\lim_{h\to 0}\dfrac{\cos\left(\frac{\pi}{6}+h\right)-\frac{\sqrt{3}}{2}}{h}$ represents the derivative of a function f at a number c. Determine f and c.

$$f(c+h)=\cos\left(\tfrac{\pi}{6}+h\right)$$
$$f(c)=\frac{\sqrt{3}}{2}$$
$$\Rightarrow f(x)=\cos x$$
$$c=\frac{\pi}{6}$$

a) $f(x)=\cos(1/6\,\pi\,x)$, $c=\dfrac{\sqrt{3}}{2}$

b) $f(x)=\cos(x)$, $c=\dfrac{\pi}{6}$

c) $f(x)=-\cos(x)$, $c=\dfrac{\sqrt{3}}{2}$

d) $f(x)=\cos(1/6\,\pi\,x)$, $c=\dfrac{\pi}{6}$

e) $f(x)=\cos(x)$, $c=\dfrac{\sqrt{3}}{2}$

C

Question 5

Given that $f(x)=6\,x^2-2\,x$ and $c=4$, find $f'(c)$ by forming the difference quotient, $\dfrac{f(c+h)-f(c)}{h}$, and taking the limit as $h\to 0$

a) 88

b) 12

c) 46

d) -2

e) 0

$$=\frac{6(4+h)^2-2(4+h)-[6(4)^2-2\cdot 4]}{h}$$
$$=\frac{6(16+8h+h^2)-8-2h-96+8}{h}$$
$$=\frac{48h+6h^2-2h}{h}$$
$$=\frac{46h+6h^2}{h}=46+6h\xrightarrow[h\to 0]{}46$$

C

Question 6

Given that $f(x)=-2\,x^2-3\,x$, find $f'(x)$ by forming the difference quotient, $\dfrac{f(x+h)-f(x)}{h}$, and taking the limit as $h\to 0$

a) 0

b) $f'(x)$ does not exist.

c) $-4\,x-3$

d) $4\,x+3$

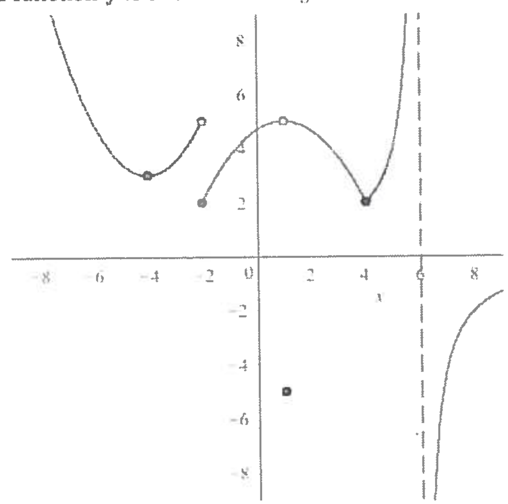
e) $4\,x-3$

$$=\frac{-2(x+h)^2-3(x+h)-(-2x^2-3x)}{h}$$
$$=\frac{-2(x^2+2xh+h^2)-3x-3h+2x^2+3x}{h}$$
$$=\frac{-2x^2-4xh-2h^2-3x-3h+2x^2+3x}{h}$$
$$=\frac{-4xh-3h-2h^2}{h}=-4x-3-2h\to -4x-3$$
as $h\to 0$

C

Question 7

The graph of a function f is shown in the figure.



At which numbers c is f continuous but not differentiable?

a) Every where differentiable

b) At $c = -4$, c = -2, c = 1 and c = 7

c) At $c = 4$

d) At $c = 1$

e) At $c = -4$

$X=-2 \Rightarrow$ Jump discontinuity

$X=1 \Rightarrow$ Removable disconti.

$X=4 \Rightarrow$ continuous but not differentiable (cusp)

$X=6 \Rightarrow$ Infinite disconti.

Question 8

Given that

$$f(x) = \begin{cases} 2x & x < -1 \\ -x^2 - 1 & x \geq -1 \end{cases}$$

and c = -1, find $f'(c)$, if it exists.

a) 3

b) -1

c) 2

d) 1

e) $f'(-1)$ does not exist

Check two limits

(a) $\displaystyle\lim_{h\to 0^+} \frac{f(-1+h)-f(-1)}{h} = \lim_{h\to 0^+} \frac{-(h-1)^2-1-[-(-1)^2-1]}{h} = \lim_{h\to 0^+} \frac{-h^2+2h-1-1+2}{h} = \lim_{h\to 0^+}(-h+2) = 2$

(b) $\displaystyle\lim_{h\to 0^-} \frac{f(-1+h)-f(-1)}{h} = \lim_{h\to 0^-} \frac{2(-1+h)+2}{h} = \lim_{h\to 0^-} 2 = 2$

(a) = (b) ✓ $f'(-1)$ exists which is 2

Question 9

Determine the values of the constants B and C so that the function given below is differentiable.

9. First, Check continuity

① $\displaystyle\lim_{x\to 1^+} f(x) = B+C$ ② $\displaystyle\lim_{x\to 1^-} f(x) = 9\cdot 1 = 9$ ③ $f(1) = 9$.

①=②=③ $\Rightarrow B+C=9$ —— (*)

$$f(x) = \begin{cases} 9\,x^2 & x \leq 1 \\ Bx + C & x > 1 \end{cases}$$

Then, check differentiability

(a) $\displaystyle\lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \frac{B(1+h)+C-9}{h} = \lim_{h\to 0^+} \frac{Bh+B+C-9}{h} \overset{\text{by } (*)}{=} \lim_{h\to 0^+} \frac{Bh}{h} = B.$

(b) $\displaystyle\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-} \frac{9(1+h)^2-9}{h} = \lim_{h\to 0^-} 18+9h = 18$

(a) = (b) $\Rightarrow B = 18$

$\boxed{\text{then by (*) } C=-9}$

a) $\{B = -18, C = 27\}$

b) $\{B = 18, C = -9\}$

c) $\{B = -36, C = -9\}$

d) $\{B = 18, C = 36\}$

e) $\{B = 18, C = 45\}$

Question 10

If $f(-4) = 2$ and $f'(-4) = -6$, find the equation of the tangent line to f at $x = -4$.

a) $y = -6\,x - 22$

b) $y = -6\,x + 2$

c) $y = 2\,x - 6$

d) $y = 2\,x + 2$

e) $y = -6\,x + 26$

$f(-4)=2 \Rightarrow$ means this line goes cross point $(-4,2)$

$f'(-4)=-6$ means the slope of ~~tangent~~ tangent line at $x=4$ is -6

By line formula $(y-y_0) = m(x-x_0)$

$\Rightarrow y-2 = -6(x+4)$

$\Rightarrow y = -6x \cancel{+12} -24 + 2$

$\Rightarrow y = -6x \cancel{+14} -22$