$* \lim_{x \to 0} \frac{\sin(ax)}{ax} = 1 \quad , \quad \lim_{x \to 0} \frac{\cos x - 1}{x} = 0$

PRINTABLE VERSION

Quiz 5

Sol.

Question 1

B

$\lim_{x \to 0} \frac{3x}{\sin(2x)} = \lim_{x \to 0} \frac{3x}{\sin(2x)} \cdot \frac{2x}{2x} = \lim_{x \to 0} \frac{2x}{\sin(2x)} \cdot \frac{3x}{2x}$

$= \left[\lim_{x \to 0} \frac{2x}{\sin(2x)}\right]\left[\lim_{x \to 0} \frac{3x}{2x}\right]$

$= 1 \cdot \frac{3}{2} = \frac{3}{2}$

a) The limit does not exist.

b) $\frac{3}{2}$

c) $\frac{2}{3}$

d) 1

e) 0

Question 2

A

$\lim_{x \to 0} \frac{\sin(3x)}{\sin(5x)} = \lim_{x \to 0} \frac{\sin(3x)}{\sin(5x)} \cdot \frac{5x}{5x} \cdot \frac{3x}{3x}$

$= \lim_{x \to 0} \frac{5x}{\sin(5x)} \cdot \frac{\sin(3x)}{3x} \cdot \frac{3x}{5x}$

$= \left[\lim_{x \to 0} \frac{5x}{\sin(5x)}\right]\left[\lim_{x \to 0} \frac{\sin(3x)}{3x}\right] \cdot \left[\lim_{x \to 0} \frac{3x}{5x}\right]$

$= 1 \cdot 1 \cdot \frac{3}{5}$

a) $\frac{3}{5}$

b) $\frac{5}{3}$

c) 0

d) The limit does not exist.

e) 1

Question 3

B

$\lim_{x \to 0} \frac{\sin(2x^2)}{6x^2} = \lim_{x \to 0} \frac{\sin(2x^2)}{6x^2} \cdot \frac{2x^2}{2x^2} = \lim_{x \to 0} \frac{\sin(2x^2)}{2x^2} \cdot \frac{2x^2}{6x^2}$

$= 1 \cdot \frac{2}{6} = \frac{1}{3}$

a) 3

b) $\frac{1}{3}$

c) 1

d) The limit does not exist.

e) 0

Question 4

B

$\lim_{x \to 0} \frac{6x}{\tan(3x)} = \lim_{x \to 0} \frac{6x}{\frac{\sin(3x)}{\cos(3x)}} = \lim_{x \to 0} \frac{6x}{\sin(3x)} \cdot \cos(3x) \cdot \frac{3x}{3x}$

$= \lim_{x \to 0} \frac{3x}{\sin(3x)} \cos(3x) \cdot \frac{6x}{3x}$

$= \left[\lim_{x \to 0} \frac{3x}{\sin(3x)}\right] \lim_{x \to 0} \left[\cos(3x)\right] \cdot \frac{6}{3}$

$= 1 \cdot 1 \cdot 2 = 2.$

a) 0

b) 2

c) $\frac{1}{2}$

d) The limit does not exist.

e) 1

Question 5

A

$$\lim_{x \to 0} \frac{\cos(3x) - 1}{5x} = \lim_{x\to 0} \frac{\cos(3x)-1}{5x} \cdot \frac{3x}{3x} = \lim_{x\to 0} \frac{\cos(3x)-1}{3x} \cdot \frac{3x}{5x}$$

$$= 0 \cdot \frac{3}{5} = 0$$

a) 0

b) $-\frac{3}{5}$

c) $\frac{5}{3}$

d) The limit does not exist.

e) 1

Question 6

B

$$\lim_{x \to 0} \frac{3x^2}{1 - \cos(4x)} = \lim_{x\to 0} \frac{3x^2}{1-\cos(4x)} \cdot \frac{4x}{4x} = \lim_{x\to 0} \frac{4x}{1-\cos(4x)} \cdot \frac{3x^2}{4x}$$

$$= \lim_{x\to 0} \frac{4x}{1-\cos(4x)} \cdot \frac{3x}{4}$$

$$= 0 \cdot 0 = 0$$

a) The limit does not exist.

b) 0

c) $\frac{\sqrt{3}}{4}$

d) $\frac{3}{8}$

e) $\frac{4}{3}$

Question 7

$\sec^2(3x) = 1 + \tan^2(3x)$

B

$$\lim_{x \to 0} \frac{1 - \sec^2(3x)}{(6x)^2} = \lim_{x\to 0} \frac{-\tan^2(3x)}{(6x)^2} = \lim_{x\to 0} \frac{-\sin^2(3x)}{(6x)(6x)} \cdot \frac{1}{\cos^2(3x)}$$

$$= \lim_{x\to 0} \frac{-\sin(3x)\cdot\sin(3x)}{(6x)(6x)} \cdot \frac{1}{\cos^2(3x)} \cdot \frac{(3x)(3x)}{(3x)(3x)}$$

$$= \lim_{x\to 0} \frac{-\sin(3x)\cdot\sin(3x)}{(3x)(3x)} \cdot \frac{1}{\cos^2(3x)} \cdot \frac{(3x)(3x)}{(6x)(6x)}$$

$$= -1 \cdot 1 \cdot 1 \cdot \frac{1}{2} \cdot \frac{1}{2} = -\frac{1}{4}$$

a) $\frac{1}{4}$

b) $-\frac{1}{4}$

c) 1

d) $-\frac{1}{2}$

e) The limit does not exist.

Question 8

B

$$\lim_{x \to 0} \frac{6}{7x\csc(6x)} = \lim_{x\to 0} \frac{6}{7x \cdot \frac{1}{\sin(6x)}} = \lim_{x\to 0} \frac{6 \cdot \sin(6x)}{7x} \cdot \frac{6x}{6x}$$

$$= \lim_{x\to 0} \frac{\sin(6x)}{6x} \cdot \frac{6 \cdot 6x}{7x}$$

$$= 1 \cdot \frac{36}{7} = \frac{36}{7}$$

a) $\frac{7}{6}$

b) $\frac{36}{7}$

c) 1

d) $-\frac{36}{7}$

e) The limit does not exist.

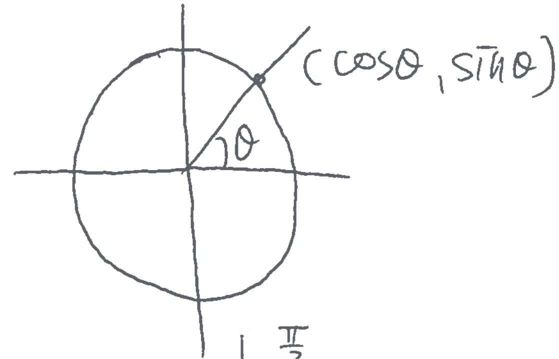
Question 9

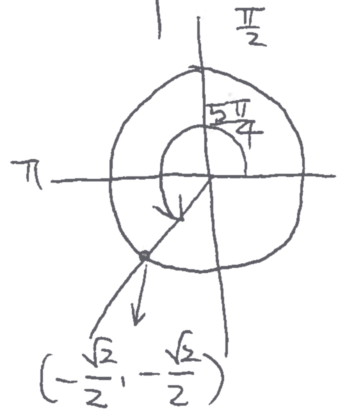
C

$\lim_{x \to \frac{5\pi}{4}} \frac{\sin(x)}{7x} =$ $\frac{\sin(\frac{5\pi}{4})}{7\cdot\frac{5\pi}{4}} = \frac{-\frac{\sqrt{2}}{2}}{\frac{35\pi}{4}}$

$= -\frac{\sqrt{2}}{2}\cdot\frac{4}{35\pi}$

$= \frac{-2\sqrt{2}}{35\pi}$





a) 1

b) The limit does not exist.

c) $\frac{-2\sqrt{2}}{35\pi}$

d) $\frac{-2\sqrt{2}}{25\pi}$

e) $\frac{2\sqrt{2}}{35\pi}$

C

Question 10

Given $f(x) = 5\cos(x)$ and $c = \frac{7\pi}{6}$, determine whether

$\lim_{h\to 0} \frac{f(c+h) - f(c)}{h}$ exists, and give the value if the limit does exist.

$$\lim_{h\to 0}\frac{5\cos(\frac{7\pi}{6}+h) - 5\cos(\frac{7\pi}{6})}{h} = \lim_{h\to 0}\frac{\frac{5}{2}\sin(h) - \frac{5\sqrt{3}}{2}\cosh + \frac{5\sqrt{3}}{2}}{h} = \lim_{h\to 0}\frac{\frac{5}{2}\sin(h)}{h} - \frac{5\sqrt{3}}{2}\underbrace{\lim_{h\to 0}\left(\frac{\cosh - 1}{h}\right)}_{0}$$

$$= \frac{5}{2}\cdot 1 = \frac{5}{2}$$

$$\boxed{\begin{aligned}\cos(\tfrac{7\pi}{6}+h) &= \cos(\tfrac{7\pi}{6})\cosh - \sin(\tfrac{7\pi}{6})\sin(h)\\ &= -\tfrac{\sqrt{3}}{2}\cosh - (-\tfrac{1}{2})\sinh\\ &= \tfrac{1}{2}\sin(h) - \tfrac{\sqrt{3}}{2}\cosh\end{aligned}}$$

a) $\frac{15}{2}$

b) $-\frac{5}{2}$

c) $\frac{5}{2}$

d) The limit does not exist.

e) $\frac{5}{4}$