Integral $\int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} + C, & n \neq -1; \\ \ln|x| + C, & n = -1. \end{cases}$

PRINTABLE VERSION

Sol

Quiz 25

Question 1

Calculate the indefinite integral: $\int \frac{6}{x^2}\,dx$. $= -\frac{6}{x} + C$

a) $-\frac{1}{x} + C$

b) $-\frac{6}{x} + C$

c) $-\frac{2}{x^3} + C$

d) $-\frac{12}{x^3} + C$

e) $-\frac{3}{x^2} + C$

Question 2

Calculate the indefinite integral: $\int \frac{3x^3 - 6}{x^2}\,dx$. $= \int \frac{3x^3}{x^2} - \frac{6}{x^2}\,dx$

$= \int 3x - \frac{6}{x^2}\,dx$

$= \frac{3}{2}x^2 + \frac{6}{x} + C$

a) $x^3 - 6x + C$

b) $\frac{3}{2}x^2 + \frac{6}{x} + C$

c) $\frac{3}{2}x^2 - 6x + C$

d) $9 - \frac{6x^3 - 12}{x^3} + C$

e) $3x + \frac{6}{x} + C$

Question 3

$5x^{\frac{1}{2}}$ x^{-3}

Calculate the indefinite integral: $\int \left(2x^3 + 5\sqrt{x} + \frac{1}{x^3}\right)dx$.

$= \frac{2x^4}{4} + 5 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{-2}}{-2} + C$

$= \frac{x^4}{2} + \frac{10}{3}x^{\frac{3}{2}} - \frac{1}{2x^{-2}} + C$

a) $\frac{2}{3}x^3 - \frac{10}{3}x^{3/2} - \frac{1}{2x^2} + C$

b) $\frac{1}{2}x^4 + \frac{10}{3}x^{3/2} - \frac{1}{2x^2} + C$

c) $\frac{1}{2}x^4 + \frac{10}{3}x^{3/2} - \frac{1}{x} + C$

d) $6x^2 + \frac{5}{2\sqrt{x}} - \frac{3}{x^4} + C$

e) $\frac{1}{2}x^4 - \frac{10}{3}x^{3/2} - \frac{1}{2x^2} + C$

Question 4

$6x^{\frac{1}{2}}$ $-x^{-\frac{1}{2}}$

Calculate the indefinite integral: $\int \left(6\sqrt{x} - \frac{1}{\sqrt{x}} + 5e^x\right)dx$.

$= 6 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 5e^x + C.$

$= 4x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + 5e^x + C.$

a) $4x^{3/2} + \sqrt{x} + \frac{1}{5}e^x + C$

b) $4x^{3/2} - 2\sqrt{x} + 5e^x + C$

c) $9x^{3/2} - 2\sqrt{x} + 5e^x + C$

d) $9x^{3/2} + 2\sqrt{x} + 5e^x + C$

e) $4x^{3/2} + 2\sqrt{x} + 5e^x + C$

Question 5

Find f givent that $f'(x) = 2x - 7$ and $f(1) = -1$.

By ①, $f(x) = \int 2x - 7\,dx = x^2 - 7x + C$.

By ② $f(1) = 1 - 7 + C = -1$.

$\Rightarrow C = 5$

$\Rightarrow f(x) = x^2 - 7x + 5$

a) $f(x) = x^2 - 7x + 1$

b) $f(x) = x^2 - 7x + 5$

c) $f(x) = 2x - 2$

d) $f(x) = 2x + 2$

e) $f(x) = x^2 - 7x + 9$

Question 6

Find f givent that $f'(x) = -5\sin(x)$ and $f(\pi) = -3$.

By ①.

$f(x) = \int f'(x)dx = -5\int \sin(x)dx$

$= -5(-\cos(x)) + C$

$= 5\cos(x) + C$.

By ② $f(\pi) = 5\cos(\pi) + C = -3$.

$\Rightarrow -5 + C = -3$,

$\Rightarrow C = 2. \Rightarrow f(x) = 5\cos(x) + 2$.

a) $f(x) = 5\cos(x) - 1$

b) $f(x) = 5\sin(x) + 3$

c) $f(x) = 5\cos(x) + 2$

d) $f(x) = 5\cos(x) + 5$

e) $f(x) = -5\sin(x) - 3$

$f'(x) = \int f''(x)dx$. — (I)

$f(x) = \int f'(x)dx$. — (II)

Question 7

Find $f(x)$ based on the following information:
$f''(x) = \sin(x)$ with $f'(Pi) = 5$ and $f(0) = 2$.

By (I), $f'(x) = \int \sin(x)dx$

$= -\cos(x) + C_1$

$f'(\pi) = -\cos(\pi) + C_1 = 5$

$\Rightarrow C_1 = 4$.

By (II), $f(x) = \int -\cos(x) + 4\,dx$.

$= -\sin(x) + 4x + C_2$

$f(0) = -\sin(0) + 4\cdot 0 + C_2 = 2$.

$\Rightarrow C_2 = 2$

$\boxed{\Rightarrow f(x) = -\sin(x) + 4x + 2.}$

a) $f(x) = -\cos(x) + 3$

b) $f(x) = -\sin(x) + 4x + 2$

c) $f(x) = \sin(x) - 4x - 1$

d) $f(x) = \cos(x) - 3$

e) $f(x) = -\sin(x) + 4x + 1$

Question 8

Calculate the indefinite integral: $\int \frac{1}{x^2+1}\,dx$.

$\arctan(x) + C$.

a) $\tan(x) + C$

b) $\arcsin(x) + C$

c) $-\frac{2x}{(x^2+1)^2} + C$

d) $\arctan(x) + C$

e) $\frac{x^2(x^2+2)}{4} + C$

Question 9

Calculate the indefinite integral: $\int \left(4\sinh(x) + x^7\right) dx$.

$= 4\cosh(x) + \frac{x^8}{8} + C$

a) $4\cosh(x) + \frac{7}{8}x^8 + C$

b) $-4\cosh(x) - \frac{1}{8}x^8 + C$

c) $4\cosh(x) + \frac{1}{8}x^8 + C$

d) $4\cosh(x) + 7x^6 + C$

e) $4\cosh(x) + \frac{1}{7}x^7 + C$

Question 10

Calculate the indefinite integral: $\int \left(\frac{1}{x} - \frac{1}{x^2} + \frac{2}{x^3}\right) dx$.

a) $\frac{1}{x} + \frac{1}{2x^2} - \frac{2}{3x^3} + C$

b) $\ln(x) + \frac{1}{x} - \frac{1}{x^2} + C$

c) $-\frac{1}{x^2} + \frac{2}{x^3} - \frac{6}{x^4} + C$

d) $\ln(x) - \frac{2}{x} - \frac{3}{x^2} + C$

e) $\ln(x) - \frac{1}{x} + \frac{1}{x^2} + C$

$= \int \frac{1}{x} - x^{-2} + 2x^{-3}\, dx$

$= \ln|x| - \frac{x^{-1}}{-1} + 2\frac{x^{-2}}{-2} + C$

$= \ln|x| + x^{-1} - x^{-2} + C$

$= \ln|x| + \frac{1}{x} - \frac{1}{x^2} + C$