Review: $(x^n)' = n x^{n-1} \quad \forall n \neq 0.$

Integral $\int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} + C, & n \neq -1; \\ \ln|x| + C, & n = -1. \end{cases}$ (Given n)

PRINTABLE VERSION

Sol

Quiz 24

Question 1

Evaluate the definite integral: $\int_0^1 (3x - 6)\,dx$

$= 3\cdot\frac{x^2}{2} - 6x \Big|_0^1$

$= \frac{3}{2}(1^2 - 0^2) - 6(1 - 0)$

$= \frac{3}{2} - 6 = -\frac{9}{2}$

a) 3

b) $-\frac{21}{2}$

c) $-\frac{9}{2}$

d) -3

e) $-\frac{15}{2}$

Question 2

Evaluate the definite integral: $\int_1^4 2\sqrt{x}\,dx = \int_1^4 2x^{\frac{1}{2}}dx$

$= 2\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \Big|_1^4$

$= \frac{4}{3} x^{\frac{3}{2}} \Big|_1^4 = \frac{4}{3}(4^{\frac{3}{2}} - 1^{\frac{3}{2}})$

$= \frac{4}{3}\cdot(8-1) = \frac{4}{3}\cdot 7 = \frac{28}{3}$

a) $\frac{62}{5}$

b) $\frac{28}{3}$

c) $\frac{124}{5}$

d) 4

e) 2

Question 3

Evaluate the definite integral: $\int_{-2}^{0} (x+6)(x-8)\,dx = \int_{-2}^{0}(x^2 - 2x - 48)dx$

$= \frac{x^3}{3} - 2\frac{x^2}{2} - 48x \Big|_{-2}^{0}$

$= \frac{0^3 - (-2)^3}{3} - (0^2 - (-2)^2) - 48(0 - (-2))$

$= \frac{8}{3} + 4 - 96 = \frac{8}{3} - 92 = -\frac{268}{3}$

a) $-\frac{292}{3}$

b) $-\frac{268}{3}$

c) $\frac{296}{3}$

d) -8

e) $\frac{152}{3}$

Question 4

Evaluate the definite integral: $\int_0^1 \left(9x^{3/4} - 10\sqrt{x}\right)dx = \int_0^1 9x^{\frac{3}{4}} - 10x^{\frac{1}{2}}dx$

$= 9\frac{x^{\frac{7}{4}}}{\frac{7}{4}} - 10\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \Big|_0^1$

$= 9\cdot\frac{4}{7}x^{\frac{7}{4}} - 10\cdot\frac{2}{3}x^{\frac{3}{2}} \Big|_0^1$

$= \frac{36}{7}(1^{\frac{7}{4}} - 0^{\frac{7}{4}}) - \frac{20}{3}(1^{\frac{3}{2}} - 0^{\frac{3}{2}})$

$= \frac{36}{7} - \frac{20}{3} = \frac{-32}{21}$

a) $\frac{248}{21}$

b) -1

c) $-\frac{32}{21}$

d) 0

e) $\frac{8}{15}$

Question 5

Evaluate the definite integral: $\int_1^2 4x\left(x^2+3\right)dx$ $= \int_1^2 4x^3 + 12x\,dx$

$= 4\frac{x^4}{4} + 12\frac{x^2}{2}\Big|_1^2 = x^4 + 6x^2\Big|_1^2$

$= (2^4 - 1^4) + 6(2^2 - 1^2).$

$= 15 + 18 = 33$

a) 40

b) $\frac{264}{5}$

c) -3

d) 33

e) $\frac{64}{3}$

$* \ (\tan(x))' = \sec^2(x) \Rightarrow \int \sec^2(x)dx = \tan(x) + C$

Question 6

Evaluate the definite integral: $\int_0^{\frac{\pi}{4}} 7\sec^2(x)\,dx$

$= 7\tan(x)\Big|_0^{\frac{\pi}{4}}$

$= 7\left[\tan\left(\frac{\pi}{4}\right) - \tan(0)\right]$

$= 7[1 - 0] = 7$

a) -14

b) 7

c) -7

d) 14

e) $\frac{7}{2}$

Question 7

Evaluate the definite integral: $\int_0^{\frac{\pi}{3}} \left(\frac{6}{\pi} - 2\sec^2(x)\right)dx$

$= \frac{6}{\pi}x - 2\tan(x)\Big|_0^{\frac{\pi}{3}}$

$= \frac{6}{\pi}\left(\frac{\pi}{3} - 0\right) - 2\left[\tan\left(\frac{\pi}{3}\right) - \tan(0)\right]$

$= 2 - 2\left[\sqrt{3} - 0\right] = 2 - 2\sqrt{3}$

a) $14/3$

b) $-2\sqrt{3} + 2$

c) $-14/3$

d) $2\sqrt{3} + 2$

e) $2\sqrt{3} - 2$

$* \ |x-1| = \begin{cases} x-1 & ; x \geq 1. \\ -(x-1) & ; x \leq 1. \end{cases}$

Question 8

Evaluate the definite integral: $\int_{-1}^{4} |x-1|\,dx$

$= \int_{-1}^{1} -(x-1)dx + \int_1^4 x-1\,dx.$

$= \int_{-1}^{1}(1-x)dx + \int_1^4 (x-1)dx$

$= x - \frac{x^2}{2}\Big|_{-1}^{1} + \frac{x^2}{2} - x\Big|_1^4$

$= (1-(-1)) - \frac{1}{2}(1^2 - (-1)^2) + \frac{1}{2}(4^2 - 1^2) - (4-1)$

$= 2 - 0 + \frac{1}{2}\cdot 15 - 3 = \frac{13}{2}$

a) 1

b) $\frac{13}{2}$

c) $\frac{25}{2}$

d) $-\frac{85}{6}$

e) $\frac{5}{2}$

Question 9

Find $\int_0^3 f(x)\,dx$ given that $f(x) = \begin{cases} 3x+1 & 0 \le x \le 1 \\ 5-x & 1 < x \le 3 \end{cases}$

$$= \int_0^1 (3x+1)\,dx + \int_1^3 (5-x)\,dx$$

$$= \frac{3}{2}x^2 + x \Big|_0^1 + 5x - \frac{x^2}{2} \Big|_1^3$$

$$= \frac{3}{2}[1^2-0^2] + [1-0] + 5[3-1] - \frac{1}{2}[3^2-1^2]$$

$$= \frac{3}{2} + 1 + 10 - 4 = \frac{17}{2}$$

a) 1

b) $\frac{21}{2}$

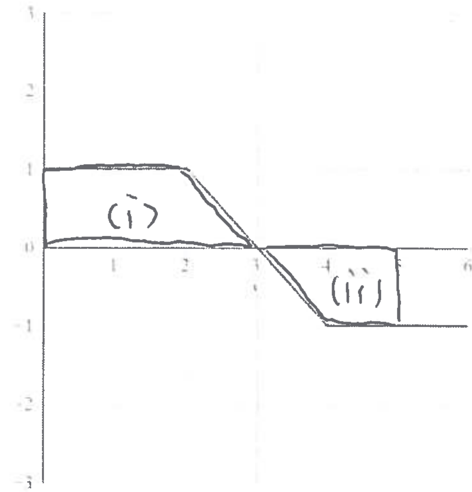
c) $\frac{17}{2}$

d) $\frac{33}{2}$

e) $-\frac{3}{2}$

Question 10

The graph of $f(x)$ is given below and $F(x) = \int_0^x f(t)\,dt$.



Find $F(5)$.

$$F(5) = \int_0^5 f(t)\,dt$$

$$= \int_0^3 f(t)\,dt + \int_3^5 f(t)\,dt$$

$$= \text{area of (i)} - \text{area of (ii)}$$

$$= 2.5 - 1.5 = 1.$$

a) 0

b) 5

c) -1

d) 1

e) 8