Given f(x)

Riemann upper sum (U_f)

$= \sum[(\text{length of subinterval}) \times (\text{max. of } f(x) \text{ on this subinterval})]$

Riemann lower sum (L_f)

$= \sum[(\text{length of subinterval}) \times (\text{min. of } f(x) \text{ on this subinterval})]$

Quiz 22

Question 1

U_f

Compute the upper Riemann sum for the given function $f(x) = x^2$ over the interval $x \in [-1, 1]$ with respect to the partition $P = \left[-1, -\frac{1}{2}, \frac{1}{2}, \frac{3}{4}, 1\right]$.

Subinterval	length	max of f
$[-1, -\frac{1}{2}]$	$\frac{1}{2}$	$f(-1) = 1$
$[-\frac{1}{2}, \frac{1}{2}]$	1	$f(-\frac{1}{2}) = f(\frac{1}{2}) = \frac{1}{4}$
$[\frac{1}{2}, \frac{3}{4}]$	$\frac{1}{4}$	$f(\frac{3}{4}) = \frac{9}{16}$
$[\frac{3}{4}, 1]$	$\frac{1}{4}$	$f(1) = 1$

a) $\frac{21}{64}$

b) $\frac{73}{64}$

c) $\frac{115}{192}$

d) $\frac{167}{192}$

e) $\frac{47}{64}$

$U_f = \frac{1}{2} \cdot 1 + 1 \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{9}{16} + \frac{1}{4} \cdot 1 = \frac{73}{64}$

Question 2

U_f

Compute the upper Riemann sum for the given function $f(x) = 2 - x^2$ over the interval $x \in [0, 1]$ with respect to the partition $P = \left[0, \frac{1}{4}, \frac{3}{4}, 1\right]$

Subinterval	length	max. of f
$[0, \frac{1}{4}]$	$\frac{1}{4}$	$f(0) = 2$
$[\frac{1}{4}, \frac{3}{4}]$	$\frac{1}{2}$	$f(\frac{1}{4}) = \frac{31}{16}$
$[\frac{3}{4}, 1]$	$\frac{1}{4}$	$f(\frac{3}{4}) = \frac{23}{16}$

a) $\frac{93}{64}$

b) $\frac{117}{64}$

$U_f = \frac{1}{4} \cdot 2 + \frac{1}{2} \cdot \frac{31}{16} + \frac{1}{4} \cdot \frac{23}{16} = \frac{117}{64}$

c) $\frac{109}{64}$

d) $\frac{105}{64}$

e) $\frac{101}{64}$

Question 3

L_f

Compute the lower Riemann sum for the given function $f(x) = \sin(x)$ over the interval $x \in [0, \pi]$ with respect to the partition $P = \left[0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi\right]$.

Subinterval	length	min of f
$[0, \frac{\pi}{6}]$	$\frac{\pi}{6}$	$\sin(0) = 0$
$[\frac{\pi}{6}, \frac{5\pi}{6}]$	$\frac{4\pi}{6}$	$\sin(\frac{\pi}{6}) = \frac{1}{2}$
$[\frac{5\pi}{6}, \pi]$	$\frac{\pi}{6}$	$\sin(\pi) = 0$

a) $\frac{5}{6}\pi$

b) $\frac{1}{3}\pi$

c) $\frac{7}{12}\pi$

d) $\frac{1}{2}\pi$

e) $\frac{2}{3}\pi$

$L_f = \frac{\pi}{6} \cdot 0 + \frac{4\pi}{6} \cdot \frac{1}{2} + \frac{\pi}{6} \cdot 0$

$= \frac{\pi}{3}$

Question 4

Estimate the integral $\int_0^6 x^2\,dx$ by the left endpoint estimate, n = 6.

$\Rightarrow f(x) = x^2$ and $P = [0, 1, 2, 3, 4, 5, 6]$

a) 55

$f(x)=x^2$

Subinterval	length	Value of left endpoint
[0,1]	1	$f(0)=0$
[1,2]	1	$f(1)=1^2=1$
[2,3]	1	$f(2)=2^2=4$
[3,4]	1	$f(3)=3^2=9$
[4,5]	1	$f(4)=4^2=16$
[5,6]	1	$f(5)=5^2=25$

b) 58

c) 52

d) 61

e) 50

Riemann Sum $=1\cdot 0+1\cdot 1+1\cdot 4+1\cdot 9+1\cdot 16+1\cdot 25$
$=55$

Question 5

Estimate the integral $\int_0^{12} 5x^2\,dx$ by the midpoint estimate, n = 6.

$f(x)=5x^2$

Subinterval	length	Value of midpoint
[0,2]	2	$f(1)=5$
[2,4]	2	$f(3)=45$
[4,6]	2	$f(5)=125$
[6,8]	2	$f(7)=245$
[8,10]	2	$f(9)=405$
[10,12]	2	$f(11)=605$

a) 2574

b) 2860

c) 3003

d) 3146

e) 2717

Riemann Sum $=2\cdot 5+2\cdot 45+2\cdot 125+2\cdot 245+2\cdot 405+2\cdot 605$
$=2860$

Question 6

Given that
$\int_0^1 f(x)\,dx=2$, $\int_0^4 f(x)\,dx=4$ and $\int_4^5 f(x)\,dx=3$ find $\int_0^5 f(x)\,dx$.

$\int_0^5 f(x)dx=\int_0^4 f(x)dx+\int_4^5 f(x)dx$
$=4+3=7$

a) 9

b) 7

c) 1

d) 4

e) 3

Question 7

Given that
$\int_0^1 f(x)\,dx=4$, $\int_0^4 f(x)\,dx=6$ and $\int_4^7 f(x)\,dx=4$ find $\int_7^1 f(x)\,dx$.

$\int_7^1 f(x)dx=-\int_1^7 f(x)dx$
$=-\left[\int_4^7 f(x)dx+\int_0^4 f(x)dx-\int_0^1 f(x)dx\right]$
$=-[4+6-4]=-6.$

a) −4

b) 0

c) 4

d) −6

e) 6

Question 8

Given that
$\int_1^4 f(x)\,dx=3$, $\int_3^4 f(x)\,dx=3$ and $\int_1^6 f(x)\,dx=7$ find $\int_4^6 f(x)\,dx$.

$\int_4^6 f(x)dx=\int_1^6 f(x)dx-\int_1^4 f(x)dx$
$=7-3=4.$

a) −4

b) 10

c) 13

d) 4

e) -7

Question 9

Given that
$\int_1^4 f(x)\,dx = 3$, $\int_3^4 f(x)\,dx = 5$ and $\int_1^7 f(x)\,dx = 6$ find $\int_3^7 f(x)\,dx$.

$$\int_3^7 f(x)dx = \int_1^7 f(x)dx - \int_1^3 f(x)dx$$
$$= \int_1^7 f(x)dx - \int_1^4 f(x)dx + \int_3^4 f(x)dx$$
$$= 6 - 3 + 5 = 8.$$

a) -8

b) -3

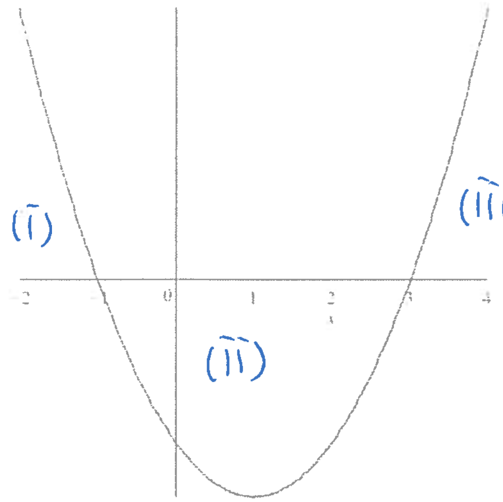
c) 8

d) 9

e) 14

Question 10

The graph of f is shown below on the interval $[-2, 4]$.



The area bounded between the graph of f and the x-axis on $[-2, -1]$ is $\frac{7}{3}$,
the area bounded between the graph of f and the x-axis on $[-1, 3]$ is $\frac{32}{3}$,
and the area bounded between the graph of f and the x-axis on $[3, 4]$ is $\frac{7}{3}$.
Determine $\int_{-2}^{-1} f(x)\,dx$. $= (i) = \frac{7}{3}$.

a) $\frac{7}{3}$

b) 0

c) $\frac{46}{3}$

d) $-\frac{7}{3}$

e) 13