Indeterminate form

$\frac{0}{0}, \frac{\infty}{\infty}$ $0\cdot\infty$, $\infty-\infty$, $0^0, 1^\infty, \infty^0$ → Take "ln", then use "L'", then Take "e"

L'

PRINTABLE VERSION

Sol

Quiz 21

Question 1

Calculate the limit: $\lim_{x\to 0}\frac{e^x-1}{10\ln(1+x)}$ $\overset{(\frac{0}{0})}{\underset{L'}{=}}\lim_{x\to 0}\frac{e^x}{\frac{10}{1+x}}$

$=\lim_{x\to 0}\frac{1}{10}e^x(1+x)=\frac{1}{10}\cdot 1\cdot 1=\frac{1}{10}$

a) -10

b) $\frac{1}{10}$

c) 1

d) 0

e) -1

Question 2

By L'Hôpital's rule

Calculate the limit: $\lim_{x\to 2}\frac{x-2}{x^2-4}$ $\overset{(\frac{0}{0})}{\underset{L'}{=}}\lim_{x\to 2}\frac{1}{2x}=\frac{1}{4}$

a) -1

b) 0

c) $-\frac{1}{4}$

d) $\frac{1}{4}$

OR

$\lim_{x\to 2}\frac{(x-2)}{(x+2)(x-2)}=\lim_{x\to 2}\frac{1}{x+2}=\frac{1}{4}$

e) 1

Question 3

Calculate the limit: $\lim_{x\to 0}\frac{3\arctan(x)}{x}$ $\overset{(\frac{0}{0})}{\underset{L'}{=}}\lim_{x\to 0}\frac{3\cdot\frac{1}{1+x^2}}{1}$

$3\arctan(0)=0$

$=\lim_{x\to 0}\frac{3}{1+x^2}=3$

a) 1

b) 0

c) -3

d) 3

e) -1

Question 4

Calculate the limit: $\lim_{x\to 0}\frac{e^x+e^{-x}-2}{1-\cos(11x)}$ $\overset{(\frac{0}{0})}{\underset{L'}{=}}\lim_{x\to 0}\frac{e^x-e^{-x}}{11\sin(11x)}$

$\overset{(\frac{0}{0})}{\underset{L'}{=}}\lim_{x\to 0}\frac{e^x+e^{-x}}{11^2\cos(11x)}=\frac{e^0+e^0}{11^2\cdot 1}=\frac{2}{121}$

a) $\frac{121}{2}$

b) $\frac{2}{121}$

c) 0

d) 1

e) $\frac{4}{121}$

Question 5

Calculate the limit: $\lim_{x\to 0}\dfrac{3+3x-3e^x}{6x(e^x-1)}$.

$\overset{(\frac{0}{0})}{\underset{L'}{=}}\lim_{x\to 0}\dfrac{3-3e^x}{6(e^x-1)+6xe^x}$

$\overset{(\frac{0}{0})}{\underset{L'}{=}}\lim_{x\to 0}\dfrac{-3e^x}{6e^x+6e^x+6xe^x}=\dfrac{-3e^0}{e^0+6e^0+0}=\dfrac{-3}{12}$

$=-\dfrac{1}{4}$

a) 0

b) $-\dfrac{1}{4}$

c) $-\dfrac{1}{2}$

d) -4

e) $\dfrac{1}{4}$

Question 6

Calculate the limit: $\lim_{x\to 0}\dfrac{xe^{9x}-x}{1-\cos(9x)}$.

$\overset{(\frac{0}{0})}{\underset{L'}{=}}\lim_{x\to 0}\dfrac{e^{9x}+9xe^{9x}-1}{-9\sin(9x)}$

$\overset{(\frac{0}{0})}{\underset{L'}{=}}\lim_{x\to 0}\dfrac{9e^{9x}+9e^{9x}+9xe^{9x}}{81\cos(9x)}$

$=\dfrac{9e^0+9e^0+0}{81\cos(0)}=\dfrac{9+9}{81}=\dfrac{2}{9}$

a) $\dfrac{2}{9}$

b) $\dfrac{9}{2}$

c) $\dfrac{4}{9}$

d) 0

e) 9

Question 7

Calculate the limit: $\lim_{x\to 0}\dfrac{4x-\sin(\pi x)}{2x^2-1}$.

$=\dfrac{0-0}{2\cdot 0-1}=\dfrac{0}{-1}=0$

a) 2

b) 1

c) 2π

d) -1

e) 0

Question 8

Calculate the limit: $\lim_{x\to\infty}\dfrac{\ln(x^7)}{x}$.

$\overset{(\frac{\infty}{\infty})}{\underset{L'}{=}}\lim_{x\to\infty}\dfrac{\frac{7}{x}}{1}$

$=\lim_{x\to\infty}\dfrac{7}{x}=0$

a) 1

b) $\dfrac{1}{7}$

c) -1

d) 7

e) 0

Question 9

Calculate the limit: $\lim_{x\to 1}x^{\left(\frac{1}{x-1}\right)}$. $\Rightarrow 1^{\infty}\Rightarrow$ use L'

Q9. $\lim_{x\to 1} x^{\left(\frac{8}{x-1}\right)} = \lim_{x\to 1} e^{\ln(x)^{\left(\frac{8}{x-1}\right)}} = \lim_{x\to 1} e^{\frac{8}{x-1}\cdot \ln x}$

[$\because e^{\ln f(x)} = f(x)$]

$= e^{\lim_{x\to 1} \frac{8}{x-1}\cdot \ln x}$

[$\because$ exp. function is continuous]

$\lim_{x\to 1} \frac{8\ln x}{x-1} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 1} \frac{\frac{8}{x}}{1} = 8$

$= e^8$

a) 0

b) $-e^8$

c) e^8

d) $e^{\frac{8}{x}}$

e) 1

Question 10

Calculate the limit: $\lim_{x\to 0}\left(\frac{8}{x} - 8\cot(x)\right)$.

$\overset{(\infty-\infty)}{=} \lim_{x\to 0}\left(\frac{8}{x} - \frac{8\cos(x)}{\sin(x)}\right)$ Combine

$= \lim_{x\to 0}\left(\frac{8\sin(x) - 8x\cos(x)}{x\sin(x)}\right) \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 0} \frac{8\cos(x) - 8\cos(x) + 8x\sin(x)}{\sin(x) + x\cos(x)}$

$\overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 0} \frac{8\sin(x) + 8x\cos(x)}{\cos(x) + \cos(x) - x\sin(x)}$

$= \frac{0}{2} = 0$

a) 2

b) 4

c) 0

d) 8

e) −8

Question 11

Calculate the limit: $\lim_{x\to\infty}\left(x^6+1\right)^{\frac{1}{\ln(x)}}$.

$\searrow \infty^0$

[$\because e^{\ln f(x)} = f(x)$]

Q11. $\lim_{x\to\infty} (x^6+1)^{\frac{1}{\ln(x)}} = \lim_{x\to\infty} e^{\ln(x^6+1)^{\frac{1}{\ln(x)}}}$

$= \lim_{x\to\infty} e^{\frac{1}{\ln(x)}\ln(x^6+1)}$

$= e^{\lim_{x\to\infty} \frac{\ln(x^6+1)}{\ln(x)}}$

[exp. function is continuous] $= e^6$

$\lim_{x\to\infty} \frac{\ln(x^6+1)}{\ln(x)} \overset{(\frac{\infty}{\infty})}{\underset{L'}{=}} \lim_{x\to\infty} \frac{\frac{6x^5}{x^6+1}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{6x^6}{x^6+1} = 6$

a) e^7

b) 0

c) e^6

d) $-e^7$

e) $-e^6$

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