Differential estimation: ① Find $f(x)$, $f'(x)$ ② pick up a ③ Find h by "a" & "$a+h$" (given) ④ $f(a+h) \approx f(a) + f'(a)\cdot h$

PRINTABLE VERSION

Sol

Quiz 20

Question 1

Use differentials to estimate the value $\sqrt[3]{25}$.

① $f(x) = \sqrt[3]{x}$ "$x^{\frac{1}{3}}$", $f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3}\frac{1}{(\sqrt[3]{x})^2}$

② $a = 27$ ($\because f(27) = 3$)

③ $a+h = 25$, $a = 27 \Rightarrow h = -2$

④ $\sqrt[3]{25} = f(25) \approx f(27) + f'(27)\cdot h$

$= 3 + \frac{1}{3}\cdot\frac{1}{3^2}\cdot -2$

$= 3 - \frac{2}{27} = \frac{79}{27}$

a) $\frac{185}{54}$

b) 1

c) $\frac{83}{27}$

d) $\frac{79}{27}$

e) $\frac{131}{54}$

Question 2

Use differentials to estimate the value $(15.5)^{1/4}$.

① $f(x) = x^{\frac{1}{4}}$, $f'(x) = \frac{1}{4}x^{-\frac{3}{4}} = \frac{1}{4}\frac{1}{(\sqrt[4]{x})^3}$

② $a = 16$ ($\because f(16) = 16^{\frac{1}{4}} = 2$)

③ $a+h = 15.5$, $a = 16 \Rightarrow h = -0.5 = -\frac{1}{2}$

④ $(15.5)^{\frac{1}{4}} = f(15.5) \approx f(16) + f'(16)\cdot h$

$= 2 + \frac{1}{4}\cdot\frac{1}{2^3}\cdot(-\frac{1}{2})$

$= 2 - \frac{1}{64} = \frac{127}{64}$

a) $\frac{127}{64}$

b) $\frac{3}{2}$

c) $\frac{159}{64}$

d) $\frac{95}{64}$

e) $\frac{129}{64}$

Question 3

Use differentials to estimate the value $\cos(62°)$.

Q3. Change to radians, Then $a+h = 62° = 62\cdot\frac{\pi}{180}$

① $f(x) = \cos(x)$, $f'(x) = -\sin(x)$

② $a = \frac{\pi}{3}$ ($\because \cos(\frac{\pi}{3})$ is known)

③ $a+h = \frac{62\pi}{180}$, $a = \frac{\pi}{3} = \frac{60\pi}{180} \Rightarrow h = \frac{2\pi}{180} = \frac{\pi}{90}$

④ $\cos(62°) = f(\frac{62\pi}{180}) \approx f(\frac{\pi}{3}) + f'(\frac{\pi}{3})\cdot\frac{\pi}{90}$

$= \cos(\frac{\pi}{3}) - [\sin(\frac{\pi}{3})]\cdot\frac{\pi}{90}$

$= \frac{1}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\pi}{90}$

$= \frac{1}{2} - \frac{\sqrt{3}\pi}{180}$

a) $\frac{\sqrt{3}}{2} + \frac{1}{180}\pi$

b) $\frac{1}{2} - \frac{\sqrt{3}}{180}\pi$

c) $\frac{1}{2} - \frac{\sqrt{3}}{90}\pi$

d) $\frac{\sqrt{3}}{2} - \frac{1}{90}\pi$

e) $\frac{1}{2} + \frac{\sqrt{3}}{180}\pi$

Question 4

Taking $\ln(2) \approx 0.69$, use differentials to estimate $\ln(2.1)$.

① $f(x) = \ln x$, $f'(x) = \frac{1}{x}$

② $a = 2$.

③ $a+h = 2.1$, $a = 2 \Rightarrow h = 0.1$

④ $\ln(2.1) = f(2.1) \approx f(2) + f'(2)\cdot h$

$= \ln 2 + \frac{1}{2}\cdot 0.1$

$= 0.69 + 0.05 = 0.74$

a) 0.740

b) 0.860

c) 0.820

d) 0.790

e) 0.660

$dy = y' \cdot \Delta x$ or $df = f'(a) \cdot h$

Question 5

Find the differential dy for $y = 3\,e^x \cos(x)$ (use $h = \Delta x$).

$dy = y' \cdot \Delta x$

$= (3e^x \cos(x) - 3e^x \sin(x))\Delta x$

a) $dy = (3\,e^x \sin(x) - e^x \cos(x))\Delta x$

b) $dy = (-3\,e^x \sin(x))\Delta x$

c) $dy = (3\,e^x \cos(x))\Delta x$

d) $dy = (3\,e^x \cos(x) - 3\,e^x \sin(x))\Delta x$

e) $dy = (-3\,e^x \sin(x) - 3\,e^x \cos(x))\Delta x$

Question 6

$df \approx f(a+h) - f(a)$

Consider the function $f(x) = x^{3/4}$. Approximate the change in f as x changes from 80 to 81.

$a+h$ a

$f'(x) = \frac{3}{4} x^{-\frac{1}{4}}$

a) $\frac{109}{4}$

b) $-\frac{3}{4}$

c) $\frac{1}{4}$

d) $\frac{107}{4}$

e) $\frac{5}{4}$

Change from 80 ~~to~~ 81 $= f(80) - f(81)$

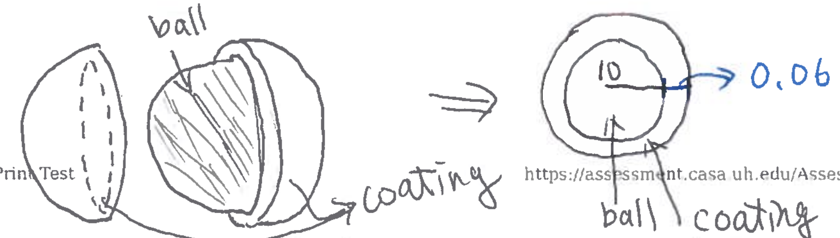
$\approx df = f'(81) \cdot \underbrace{(-1)}_{h} = \frac{3}{4} \cdot \frac{1}{\sqrt[4]{81}} \cdot (-1)$

$= \frac{3}{4} \cdot \frac{1}{3} \cdot (-1)$

$= -\frac{1}{4}$

Question 7

$\Rightarrow$ changing $|-\frac{1}{4}| = \frac{1}{4}$



A spherical ball bearing will be coated by 0.06 cm of protective coating. If the radius of this ball bearing is 10 cm, approximately how much coating will be required? (use $\pi \approx 3.14$)

① $f(x) = \frac{4}{3}\pi r^3$ (Volume of ball)

$f'(x) = 4\pi r^2$.

② $a+h = 10 + 0.06 = 10.06$.

$a = 10 \Rightarrow h = 0.06$.

③ Volume of Coating $= f(10.06) - f(10)$

$\approx df = f'(10) \cdot h$

$= 4\pi \cdot 10^2 \cdot 0.06$

$= 75.360 \text{ cm}^3$

a) 4262.026 cm^3

b) 75.360 cm^3

c) 4111.306 cm^3

d) 74.360 cm^3

e) 76.360 cm^3

Question 8

Give the derivative of $f(x) = e^{\arcsin(2x)}$ at the point where $x = \frac{1}{4}$.

$f'(x) = \frac{2}{\sqrt{1-(2x)^2}} \cdot e^{\arcsin(2x)}$

$f'(\frac{1}{4}) = \frac{2}{\sqrt{1-\frac{1}{4}}} e^{\arcsin(\frac{1}{2})}$

$= \frac{2}{\sqrt{\frac{3}{4}}} \cdot e^{\frac{\pi}{6}} = \frac{4}{\sqrt{3}} e^{\frac{\pi}{6}} = \frac{4\sqrt{3}}{3} e^{\frac{\pi}{6}}$

a) $\frac{\sqrt{3}}{3} e^{\pi/3}$

b) 1

c) 4

d) $\frac{4\sqrt{3}}{3} e^{\pi/6}$

e) $\frac{2\sqrt{3}}{3} e^{\pi/6}$

Question 9

Find the derivative of $(3x+4)^{5x}$.

Q9. let $y=(3x+4)^{5x}$.

① $\ln y = \ln (3x+4)^{5x} = 5x \ln(3x+4)$.

② $\frac{y'}{y} = 5 \cdot \ln(3x+4) + 5x \cdot \frac{3}{3x+4}$.

③ $y' = (3x+4)^{5x}\left[5 \cdot \ln(3x+4) + \frac{15x}{3x+4}\right]$

a) $15x(3x+4)^{5x-1}$

b) $(3x+4)^{5x}\left(5\ln(3x+4)+\frac{15x}{3x+4}\right)$

c) $(3x+4)^{5x}\left(5\ln(3x+4)-\frac{5}{3x+4}\right)$

d) $\left(5\ln(3x+4)+\frac{15x}{3x+4}\right)$

e) $5x(3x+4)^{5x-1}$

Question 10

Given $f(x)=3x^3+x-7$ verify that $f(x)$ is invertible and, if so, find the equation of the tangent line to $f^{-1}(x)$ at the point where $x=-33$. Note that $f(\underset{a}{\underline{-2}})=\underset{b}{\underline{-33}}$. $\Rightarrow f^{-1}(-33)=-2$.

$f'(x) = 9x^2+1$.

Tangent line:

① slope : $(f^{-1})'(-33) = \frac{1}{f'(-2)} = \frac{1}{9(-2)^2+1} = \frac{1}{37}$.

② point: $f^{-1}(-33)=-2 \Rightarrow (-33,-2)$.

by ①② $\Rightarrow (y+2) = \frac{1}{37}(x+33)$

a) $f(x)$ is not invertible.

b) $y+33=-\frac{1}{37}(x-2)$

c) $y+2=\frac{1}{37}(x+33)$

d) $y+33=\frac{1}{37}(x+2)$

e) $y-2=-\frac{1}{37}(x-33)$