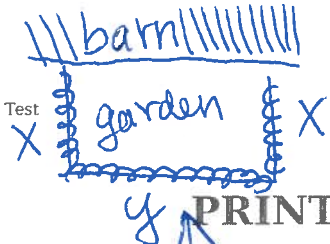


PRINTABLE VERSION

Quiz 19

Sol

Question 1

A rectangular garden 98 square feet in area is to be fenced off against rats. Find the dimensions that will require the least amount of fencing if one side of the garden is already protected by a barn.

a) 56 by $\frac{7}{4}$ feet

b) 42 by $\frac{7}{3}$ feet

c) 14 by 7 feet

d) 13 by 6 feet

e) 16 by 6 feet

(1) The min function: $2x+y$.
(2) The relation: $xy=98 \Rightarrow y=\frac{98}{x}$.
(3) restriction: $x>0,\ y>0$
(4) $f(x)=2x+y=2x+\frac{98}{x}$, $f'(x)=2-\frac{98}{x^2}=\frac{2x^2-98}{x^2}$.
Critical number: $x=7$ or $\cancel{-7}$
f': 0 $- - -$ 7 $+ + + +$ has local min
$\Rightarrow x=7$ $y=\frac{98}{7}=14$.

Question 2

Find the largest possible area for a rectangle with base on the x-axis and upper vertices on the curve $y = 9 - x^2$.

a) $12\sqrt{3}$

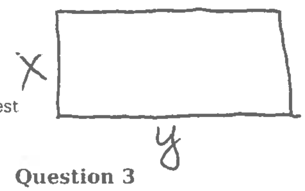
b) 36

c) $6\sqrt{3}$

d) $6\sqrt{6}$

e) $24\sqrt{3}$

See HW11, Q4.



Question 3

Of all the rectangles with an area of 25 square feet, find the dimensions of the one with the smallest perimeter.

a) 5 ft. x 5 ft.

b) $\frac{5}{2}$ ft. x 10 ft.

c) $\frac{5}{3}$ ft. x 15 ft.

d) *not possible*

e) $\frac{5}{4}$ ft. x 20 ft.

(1) The min function: $2x+2y$
(2) The relation: $xy=25 \Rightarrow y=\frac{25}{x}$
(3) The restriction: $x>0.\ y>0$
(4) $f(x)=2x+2y=2x+2\frac{25}{x}$, $f'(x)=2-\frac{50}{x^2}=\frac{2x^2-50}{x^2}$.
critical number: $x=5$ or $\cancel{-5}$
0 $- - -$ 5 $+ + + + +$ has min. $\Rightarrow x=5,\ y=5$.

Question 4

Of all the rectangles with a perimiter of 40 feet, find the dimensions of the one with the largest area.

a) *not possible*

b) 10 ft. x 10 ft.

c) 5 ft. x 15 ft.

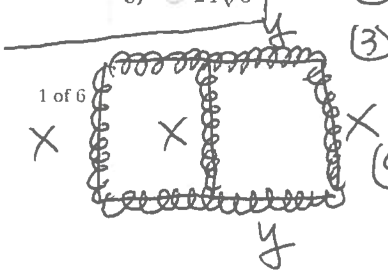
d) $\frac{10}{3}$ ft. x $\frac{50}{3}$ ft.

e) $\frac{5}{2}$ ft. x $\frac{35}{2}$ ft.

(1) The max function: xy.
(2) The relation $2x+2y=40 \Rightarrow y=20-x$
(3) The restriction: $0<x<20$.
(4) $f(x)=xy=x(20-x)$, $f'(x)=-2x+20$
Critical number: $x=10$
0 $+ + + +$ 10 $- - - -$ 20 $\Rightarrow$ local max $\Rightarrow x=10,\ y=10$

Question 5

A rectangular playground is to be fenced off and divided into two parts by a fence parallel to one side of the playground. 1200 feet of fencing is used. Find the dimensions of the playground that will enclose the greatest total



Q5 (1) The max function: xy.
(2) The relation: $3x+2y=1200 \Rightarrow y=\frac{1200-3x}{2}$.
(3) The restriction: $0<\frac{1200-3x}{2}$ and $x>0$
$\Rightarrow 0<x<400$
(4) $f(x)=xy=x\cdot\left(\frac{1200-3x}{2}\right) \Rightarrow f'(x)=-3x+600$
critical: $x=200$
0 $+ + + +$ 200 $- - - -$ 400 has max. value.
$\Rightarrow x=200,\ y=300$

area.

a) 320 by 210 feet with the divider 210 feet long

b) 300 by 300 feet with the divider 300 feet long

c) 310 by 210 feet with the divider 310 feet long

d) 300 by 200 feet with the divider 200 feet long

e) 295 by 205 feet with the divider 296 feet long

Question 6

Find A and B given that the function $y = \frac{A}{\sqrt{x}} + B\sqrt{x}$ has a minimum value of 4 at x = 1.

$\Rightarrow x=1, y=4 \Rightarrow A+B=4$ —(1)

$y' = -\frac{A}{2\sqrt{x^3}} + \frac{B}{2\sqrt{x}} = \frac{Bx-A}{2\sqrt{x^3}}$

critical number: $x=1 \Rightarrow B-A=0$ —(2)

$\Rightarrow B=A$

$\Rightarrow B=2, A=2$

a) A = 2 and B = 6

b) A = 4 and B = 4

c) A = 4 and B = 2

d) A = 2 and B = 2

e) A = 2 and B = 4

Question 7

Find the coordinates of the point(s) on the curve $3y = 18 - x^2$ that are closest to the origin.

(1) The min function is distance between (0,0) to (x,y) (which is on "3y = 18 − x²")

$\Rightarrow d^2 = (x-0)^2 + (y-0)^2 = x^2 + y^2$

a) $\left(\frac{3\sqrt{6}}{2}, \frac{3}{2}\right)$

(2) The relation: $3y = 18 - x^2 \Rightarrow x^2 = 18 - 3y$.

(3) The restriction: $y \le 6$

(4) $f(y) = x^2 + y^2 = 18 - 3y + y^2 \Rightarrow f'(y) = 2y - 3 \Rightarrow$ critical $y = \frac{3}{2}$

− − − − $\frac{3}{2}$ + + + + 6 local min $\Rightarrow y = \frac{3}{2}$ $x = \pm\frac{3\sqrt{6}}{2}$

b) $\left(\frac{3\sqrt{6}}{2}, \frac{3}{2}\right)$ and $\left(\frac{-3\sqrt{6}}{2}, \frac{3}{2}\right)$

c) $(0, 6)$, $\left(\frac{3\sqrt{6}}{2}, \frac{3}{2}\right)$ and $\left(\frac{-3\sqrt{6}}{2}, \frac{3}{2}\right)$

d) $\left(1, \frac{17}{3}\right)$

e) $\left(\frac{-3\sqrt{6}}{2}, \frac{3}{2}\right)$

Q8.

(1) The min. function.

$d^2 = (x-5)^2 + (y-0)^2$.

(2) The relation:

$y = \sqrt{x+2} \Rightarrow y^2 = x+2$.

(3) The restriction:

$x+2 \ge 0 \Rightarrow x \ge -2$

and $y \ge 0$.

Question 8

Find the coordinates of the point(s) on the curve $y = \sqrt{x+2}$ that are closest to the point (5, 0).

(4) $f(x) = (x-5)^2 + y^2 = (x-5)^2 + x + 2$

$= x^2 - 9x + 27. \Rightarrow f'(x) = 2x - 9.$

a) $(1, \sqrt{3})$

b) $\left(\frac{9}{2}, \frac{\sqrt{26}}{2}\right)$

critical number $x = \frac{9}{2}$

c) $(9, \sqrt{11})$

−2 − − − $\frac{9}{2}$ + + + + ⇒ has min.

d) $\left(\frac{11}{2}, \frac{\sqrt{30}}{2}\right)$

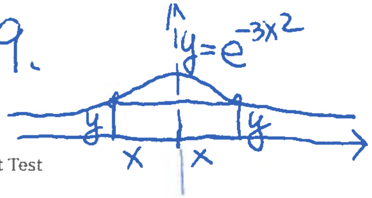
$x = \frac{9}{2}$ $y = \frac{\sqrt{26}}{2}$ or $\cancel{-\frac{\sqrt{26}}{2}}$

e) $(0, \sqrt{2})$ and $\left(\frac{9}{4}, \frac{\sqrt{17}}{2}\right)$

Question 9

A rectangle has one side on the x-axis and the upper two vertices on the graph of $y = e^{-3x^2}$. Where should the vertices be placed so as to maximize

Q9.



① The max function: $2xy$.
② The relation: $y=e^{-3x^2}$.
③ The restriction: $x>0$, $y>0$.
④ $f(x)=2xy=2xe^{-3x^2}$.

$f'(x)=2e^{-3x^2}-12x^2e^{-3x^2}$
$=(2-12x^2)e^{-3x^2}$.

$f'(x)=0 \Rightarrow 2-12x^2=0$ ($\because e^{-3x^2}>0$ $\forall x \in \mathbb{R}$)
$\Rightarrow x=\frac{\sqrt{6}}{6}$

0 ++++ $\frac{\sqrt{6}}{6}$ - - - - $\Rightarrow$ has max,

the area of the rectangle?

a) $\left(\frac{\sqrt{6}}{12}, e^{-1/2}\right)$ and $\left(-\frac{\sqrt{6}}{12}, e^{-1/2}\right)$

b) $\left(-\frac{\sqrt{6}}{6}, \frac{1}{2}e^{-1/2}\right)$ and $\left(-\frac{\sqrt{6}}{6}, \frac{1}{2}e^{-1/2}\right)$

c) $\left(\frac{\sqrt{6}}{6}, e^{-1/2}\right)$ and $\left(-\frac{\sqrt{6}}{6}, e^{-1/2}\right)$

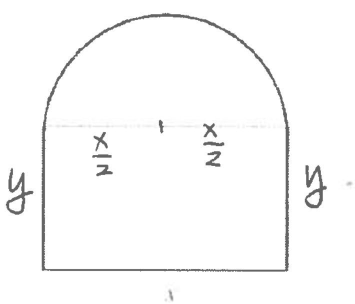
d) $\left(\frac{\sqrt{6}}{3}, e^{-1/2}\right)$ and $\left(-\frac{\sqrt{6}}{3}, e^{-1/2}\right)$

e) $\left(\frac{\sqrt{6}}{6}, -\frac{1}{2}e^{-1/2}\right)$ and $\left(-\frac{\sqrt{6}}{6}, \frac{1}{2}e^{-1/2}\right)$

$\Rightarrow$ Two points $\left\{\begin{array}{l}\left(\frac{\sqrt{6}}{6}, e^{-\frac{1}{2}}\right)\\ \left(-\frac{\sqrt{6}}{6}, e^{-\frac{1}{2}}\right)\end{array}\right.$

Question 10

The figure below shows a region that consists of a semi-circle on top of a rectangle. Give the value of x that maximizes the area of the region if the circumference of the region is 8.



a) $\frac{16\pi}{\pi-2}$

b) $\frac{8}{\pi+2}$

c) $\frac{8\pi}{\pi+4}$

d) $\frac{16}{\pi+4}$

e) $\frac{16}{\pi-2}$

area of rectangle & semi-circle.

① The max area: $xy+\left(\frac{x}{2}\right)^2\pi\cdot\frac{1}{2}$.

② The relation: $x+2y+x\cdot\pi\cdot\frac{1}{2}=8$.

length, two widths, semi-circle

$\Rightarrow 2y=8-x-\frac{\pi}{2}x$. $\Rightarrow y=4-\frac{x}{2}-\frac{\pi}{4}x$.

③ The restriction: $y>0$, $x>0$

④ $f(x)=xy+\frac{\pi}{8}x^2=x\left(4-\frac{x}{2}-\frac{\pi}{4}x\right)+\frac{\pi}{8}x^2=4x-\frac{x^2}{2}-\frac{\pi}{8}x^2$.

$f'(x)=4-x-\frac{\pi}{4}x \Rightarrow$ critical number: $x=\frac{4}{1+\frac{\pi}{4}}=\frac{16}{4+\pi}$.

0 ++++ $\frac{16}{4+\pi}$ - - - - $\Rightarrow$ local max,

$x=\frac{16}{4+\pi}$