PRINTABLE VERSION

Sol

Quiz 18

Question 1

Differentiate the given function $y = \cosh(6x^2 - 4)$.

a) $-\sinh(12x)$

b) $12x\sinh(6x^2+4)$

c) $(6x^2-4)\sinh(6x^2-4)$

d) $\sinh(6x^2+4)$

e) $-12\sinh(6x^2-4)$

$y' = (6x^2+4)'\sinh(6x^2+4)$

$= 12x \cdot \sinh(6x^2+4)$

Question 2

Differentiate the given function $y = \sqrt{\sinh(10x)}$.

a) $\frac{-10}{\sqrt{\sinh(10x)}}$

b) $\frac{-5\cosh(10x)}{\sqrt{\sinh(10x)}}$

c) $\frac{\cosh(10x)}{2\sqrt{\sinh(10x)}}$

$= (\sinh(10x))^{\frac{1}{2}}$

$y' = \frac{1}{2}(\sinh(10x))^{-\frac{1}{2}} \cdot \cosh(10x) \cdot 10$

$= \frac{10}{2} \cdot \frac{\cosh(10x)}{\sqrt{\sinh(10x)}}$

d) $\frac{10}{\sqrt{\sinh(10x)}}$

e) $\frac{5\cosh(10x)}{\sqrt{\sinh(10x)}}$

Q3.

$y' = 10\cosh(10x)\cosh(10x) + 10\sinh(10x)\sinh(10x)$

$= 10[\cosh(10x)]^2 + 10[\sinh(10x)]^2$

Question 3

Differentiate the given function $y = \sinh(10x)\cosh(10x)$.

product

a) $10\cosh(10x)]^2 - 10[\sinh(10x)]^2$

b) 1

c) 10

d) $10[\cosh(10x)]^2 + 10[\sinh(10x)]^2$

e) $10[\cosh(10x)]^2 - [\sinh(10x)]^2$

Q(4)

$(\sinh(4x))' = 4\cosh(4x)$

$(\cosh(4x) - 7)' = 4\sinh(4x)$

Question 4

quotient

Differentiate the given function $y = \frac{\sinh(4x)}{\cosh(4x) - 7}$.

a) $\frac{4 - 28\cosh(4x)}{(\cosh(4x) - 7)^2}$

b) $\frac{4}{(\cosh(4x) - 7)^2}$

c) $\frac{16 - 112\cosh(4x)}{(\cosh(4x) - 7)^2}$

$y' = \frac{4\cosh(4x)[\cosh(4x) - 7] - 4[\sinh(4x)]}{[\cosh(4x) - 7]^2}$

$= \frac{4[\cosh(4x)]^2 - 4[\sinh(4x)]^2 - 28\cosh(4x)}{[\cosh(4x) - 7]^2}$

$[\cosh(4x)]^2 - [\sinh(4x)]^2 = 1$

$= \frac{4 - 28\cosh(4x)}{[\cosh(4x) - 7]^2}$

d) $\dfrac{7}{(\cosh(4x)-7)^2}$

e) $\dfrac{1-7\cosh(4x)}{\cosh(4x)-7}$

$e^{\ln(u(x))}=u(x)$

$\sinh(x)=\dfrac{e^x-e^{-x}}{2}$, $\cosh(x)=\dfrac{e^x+e^{-x}}{2}$

Question 5

Differentiate the given function $y=\cosh\left(\ln\left(2x^4\right)\right)$.

$=\dfrac{e^{\ln 2x^4}+e^{-\ln(2x^4)}}{2}$

$=\dfrac{2x^4+e^{\ln(2x^4)^{-1}}}{2}$

a) $x^3+\dfrac{1}{x^5}$

b) $4x^3-\dfrac{2}{x^4}$

c) $x^3-\dfrac{4}{x^5}$

d) $4x^3+\dfrac{1}{x^4}$

e) $4x^3-\dfrac{1}{x^5}$

$=\dfrac{2x^4+(2x^4)^{-1}}{2}=\dfrac{2x^4}{2}+\dfrac{(2x^4)^{-1}}{2}$

$=x^4+\dfrac{1}{2}\cdot\dfrac{1}{2x^4}=x^4+\dfrac{1}{4x^4}$

$y'=4x^3-\dfrac{4}{4x^5}=4x^3-\dfrac{1}{x^5}$

Question 6

Differentiate the given function $y=\arctan(\cosh(2x))$.

a) $\dfrac{-\cosh(2x)}{(\sinh(2x))^2-1}$

b) $\dfrac{2\sinh(2x)}{1-(\cosh(2x))^2}$

$y'=\dfrac{(\cosh(2x))'}{1+(\cosh(2x))^2}$

$=\dfrac{2\sinh(2x)}{1+(\cosh(2x))^2}$

c) $\dfrac{-2\sinh(2x)}{1+(\cosh(2x))^2}$

d) $\dfrac{2\sinh(2x)}{1+\cosh(2x)}$

e) $\dfrac{\sinh(2x)}{1+(\cosh(2x))^2}$

Q7.

$y'=\dfrac{(\sinh(5x))'}{\sinh(5x)}$

$=\dfrac{5\cosh(5x)}{\sinh(5x)}$

Question 7

Differentiate the given function $y=\ln(\sinh(5x))$.

a) $\dfrac{5}{\sinh(5x)}$

b) $\dfrac{-5\cosh(5x)}{\sinh(5x)}$

c) $\dfrac{5\cosh(5x)}{\sinh(5x)}$

d) $\dfrac{\cosh(5x)}{\sinh(5x)}$

e) $\dfrac{5}{\cosh(5x)}$

Q8. Take "ln"

$\ln y=x\cdot\ln(\cosh(10x))$ $\Rightarrow$ Do derivative.

$\dfrac{y'}{y}=\ln(\cosh(10x))+x\cdot\dfrac{10\sinh(10x)}{\cosh(10x)}$

$\Rightarrow y'=(\cosh(10x))^x\left[\ln(\cosh(10x))+\dfrac{10x\sinh(10x)}{\cosh(10x)}\right]$

Question 8

Differentiate the given function $y=(\cosh(10x))^x$.

a) $(\sinh(10x))^x\left(\ln(\sinh(10x))+\dfrac{10x\cosh(10x)}{\sinh(10x)}\right)$

b) $(\cosh(10x))^x\left(\ln(\cosh(10x)) + \dfrac{10\sinh(10x)}{\cosh(10x)}\right)$

c) $(\cosh(10x))^x\left(\ln(\cosh(10x)) - \dfrac{10}{\cosh(10x)}\right)$

d) $(\sinh(10x))^x\left(\ln(\cosh(10x)) - \dfrac{10}{\cosh(10x)}\right)$

e) $(\cosh(10x))^x\left(\ln(\cosh(10x)) - \dfrac{10x\sinh(10x)}{\cosh(10x)}\right)$

Question 9

Find the absolute extreme values of $y = -\dfrac{65}{2}\cosh(x) + \dfrac{63}{2}\sinh(x)$.

$= -\frac{65}{2}\left(\frac{e^x+e^{-x}}{2}\right) + \frac{63}{2}\left(\frac{e^x-e^{-x}}{2}\right)$

$= \left(-\frac{65}{4}+\frac{63}{4}\right)e^x + \left(-\frac{65}{4}-\frac{63}{4}\right)e^{-x}$

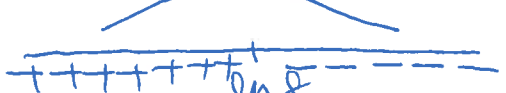
$= -\frac{1}{2}e^x - 32e^{-x}$

$y' = -\frac{1}{2}e^x + 32e^{-x} = 0 \Rightarrow \frac{e^x}{2} = \frac{32}{e^x}$

$\Rightarrow e^{2x} = 64$ Take "ln" $\Rightarrow \ln(e^{2x}) = \ln 64$

$\Rightarrow 2x = \ln 8^2 = 2\ln 8 \Rightarrow x = \ln 8$

Number line of y'



local min $\Rightarrow$ abs. min.

$y(\ln(8)) = -\frac{1}{2}e^{\ln 8} - 32e^{-\ln 8}$

$= -\frac{1}{2}\cdot 8 - 32\cdot\frac{1}{8}$

$= -4 - 4 = -8$.

a) absolute max: $f(\ln(16)) = -16$

b) absolute max: $f(\ln(8)) = \sinh(4)$

c) absolute max: $f(\ln(8)) = -8$

d) absolute min: $f(\ln(8)) = \cosh(4)$

e) absolute min: $f(\ln(8)) = -8$

Question 10

Determine A, B, and C so that $y = A\cosh(Cx) + B\sinh(Cx)$ satisfies the conditions $y'' - 9y = 0$, $y(0) = 1$, $y'(0) = 3$. Take $C > 0$.

$y' = AC\sinh(Cx) + BC\cosh(Cx)$ $\xrightarrow{y'(0)=3}$ $AC\cdot\underbrace{\sinh(0)}_{0} + BC\underbrace{\cosh(0)}_{1} = BC = 3$ — (1)

$y'' = AC^2\cosh(Cx) + BC^2\sinh(Cx)$ $\xrightarrow{y''-9y=0}$ $AC^2\cosh(Cx) + BC^2\sinh(Cx) - 9(A\cosh(Cx) + B\sinh(Cx)) = 0$ — (2)

$y = A\cosh(Cx) + B\sinh(Cx)$ $\xrightarrow{y(0)=1}$ $A\cdot\cosh(0) + B\sinh(0) = A = 1$ — (3).

From (2), we have.

$$(A^2C - 9A)\cosh(cx) + (B^2C - 9B)\sinh(cx) = 0$$

$$\Rightarrow (Ac^2 - 9A)\frac{e^x + e^{-x}}{2} + (Bc^2 - 9B)\cdot\frac{e^x - e^{-x}}{2}$$

$$\Rightarrow \left[(Ac^2 - 9A) + (Bc^2 - 9B)\right]\frac{e^x}{2} + \left[(Ac^2 - 9A) - (Bc^2 - 9B)\right]\frac{e^{-x}}{2} = 0, \ \forall x$$

$$\Rightarrow Ac^2 - 9A + (Bc^2 - 9B) = 0 \text{ and } (Ac^2 - 9A) - (Bc^2 - 9B) = 0$$

$$\Leftrightarrow Ac^2 - 9A = 0 \text{ and } Bc^2 - 9B = 0.$$

From (3), $A = 1 \Rightarrow c^2 = 9 \Rightarrow c = 3$ $(c > 0)$.

From (1), $BC = 3 \Rightarrow B = 1$.

$$\Rightarrow A = 1,\ B = 1,\ C = 3.$$